Art, Architecture and Mathematics in the Ladies Abbey Romanesque Church, Caen: a mirror as a measuring device. Written by O. Jenvrin and JP Le Goff

1   Assigment and final tasks.

The mathematical task is to measure a large distance that cannot be measured directly such as a height as in ancient times in church architecture ? You will explain a method to calculate such inaccessible distances to human size.

The communication task is a peer to peer students conference based on a self made poster to be given as experts in direction to visitors. All members of the team have to speak equally when presenting to the visitors.

The collaborative task is a team work in a foreign language and members are called below Alice, Bob, Charlie, David and Emmy. Feel free to share roles.

2   Measuring a human sized height

Alice chooses one of the round arches on both sides in the chancel. Consider the height from ground to the keystone of this round arch and Bob measures it with a simple measure tape. What is this height denoted by h, Charlie keeps this result in his notes for future comment.

Your task is to measure this height with the method below and check the result. It were used back to the Middle Age and earlier to evaluate unaccessible heights such as the height of a building which cannot be measured directly with a measure tape.
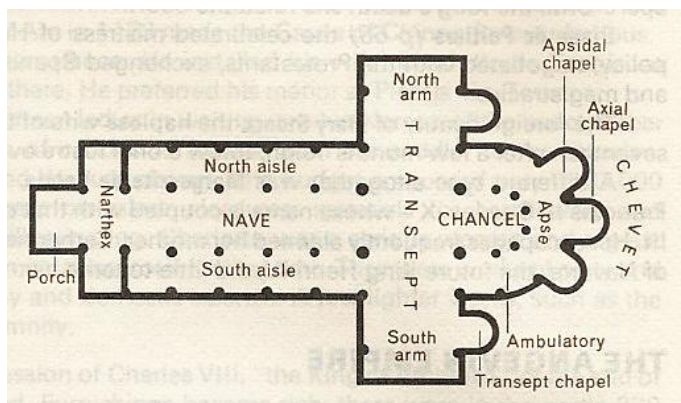
3   Experiencing the method on a human sized height
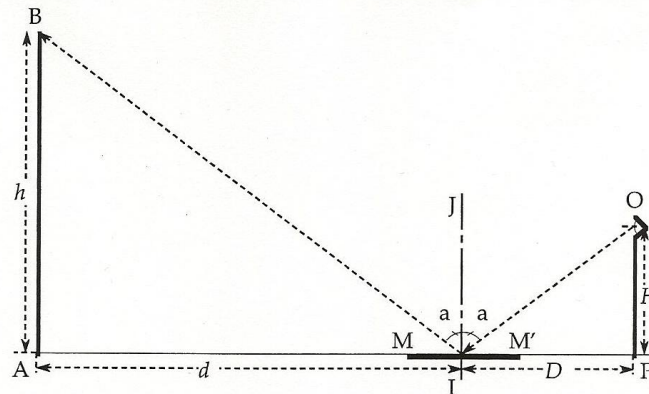
Share roles in the following instructions.

1. Let $h = AB$ denote the unknown height that has to be evaluated and let $H = OP$ denote the observer's vertical distance between eyes and ground. Lay carefully and horizontally the mirror onto the ground and walk backwards such that the observer can see point B reflecting onto the mirror precisely on its red line. STOP AND DO NOT MOVE! You are now being measured by your classmate.

2. Let point I denote the center point of the mirror. Measure and write the following distances :
   – horizontal distance $d = AI$ between the base of targeted height on the ground and the mirror,
   – horizontal distance $D = IP$ between mirror's center point and observer's feet,
   – vertical distance $H = OP$ between observer's eyes and ground floor.

3. Now remember the incidence reflection rules on angles. On a plane mirror, the incidence angle equals the reflection angle therefore triangles ABI and POI share proportional side lengths. For example, $\dfrac{h}{d} = \dfrac{H}{D}$.

4. Using this rule evaluate the distance between keystone and ground and check your answer with the real measurement. Do you think that this calculating method for distances is satisfying ?

# 4   A mirror to evaluate an unaccessible height

1. Choose a much larger round arch between the nave and aisle.

2. Consider the vertical projection onto the ground of the keystone and set a rubber to mark this point. The targeted height is the distance between the keystone and the ground.

3. Apply the method given above to evaluate this distance.

4. Give an approximate value for this unaccessible height. Does this result look correct to your eyes ? Comment.

5. Compare your findings with a group using a Gerbert stick as a device to measure the same heights as you just did. Do you find the same results ?

5   Good to know

This method was used in the Middle Age, as explained in Latin by a monk, Gerbert d'Aurillac in the XIIth century in an ancient manuscript from the "Mont Saint-Michel Abbey" which can be seen nowadays at the city library of Avranches, two hours away from Caen. In those times the mirror was a clam shell filled with water since water level is necessarily horizontal. The second diagram was sketched by Bartoli.
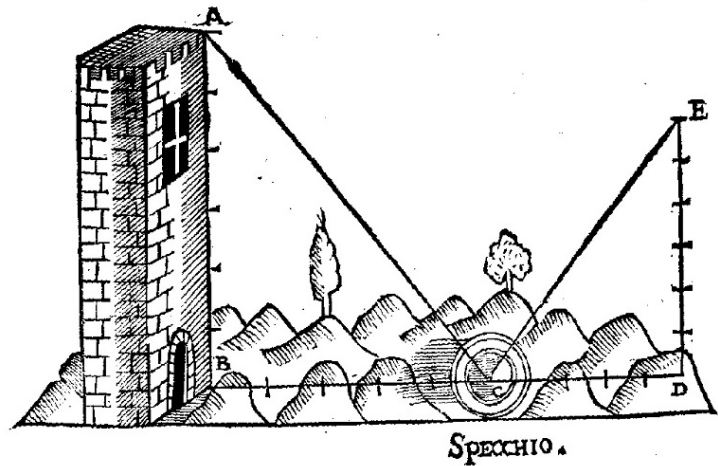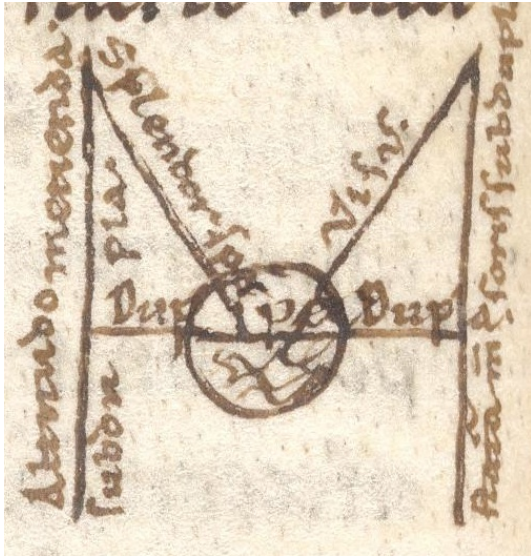
6   Bibliography

Adapted from "Finale du Rallye mathématique, vendredi 30 avril 2004, *mesure par visée de grandeurs inaccessibles*, Jacqueline Leparmentier, Didier Trotoux, Jean-Pierre Le Goff, Cercle d'histoire des sciences de l'IREM de Basse-Normandie.

7   Provided material

– a mirror marked with a red perpendicular bisector
– a double decameter measure tape, used in sport classes
– a level
– a plumb-line
– answer sheet with grid for measurements
– a protractor, set square and ruler
– blank sheets for further drawings to be glued onto the poster
– separate photos of places in the church and the church plan given in this assignment
– photos of the historical diagrams

Answer sheet to fill in the measurements, for calculations and personal notes

Name ....

Team ....

human sized round arch in the chancel : direct measurement gives..............................

measurements	human sized round arch	large round arch in the nave
$d = AI$		
$D = IP$		
$H = OP$		
$\dfrac{H}{D}$		
h		