1 Assignment and final tasks

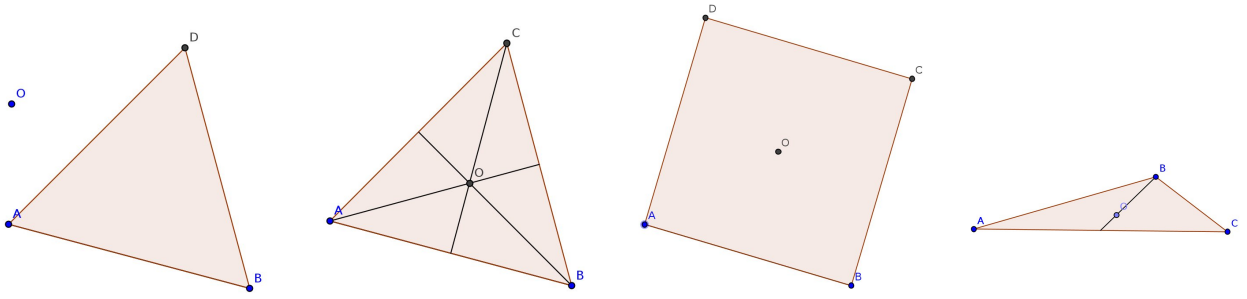
The mathematical task is to tell reasons why any decorative art piece looks beautiful in church architecture ? You will explain how this has to do with symmetries hidden in the motif.

The communication task is a peer to peer students conference based on a self made poster to be given as experts in direction to visitors. All members of the team have to speak equally when presenting to the visitors.

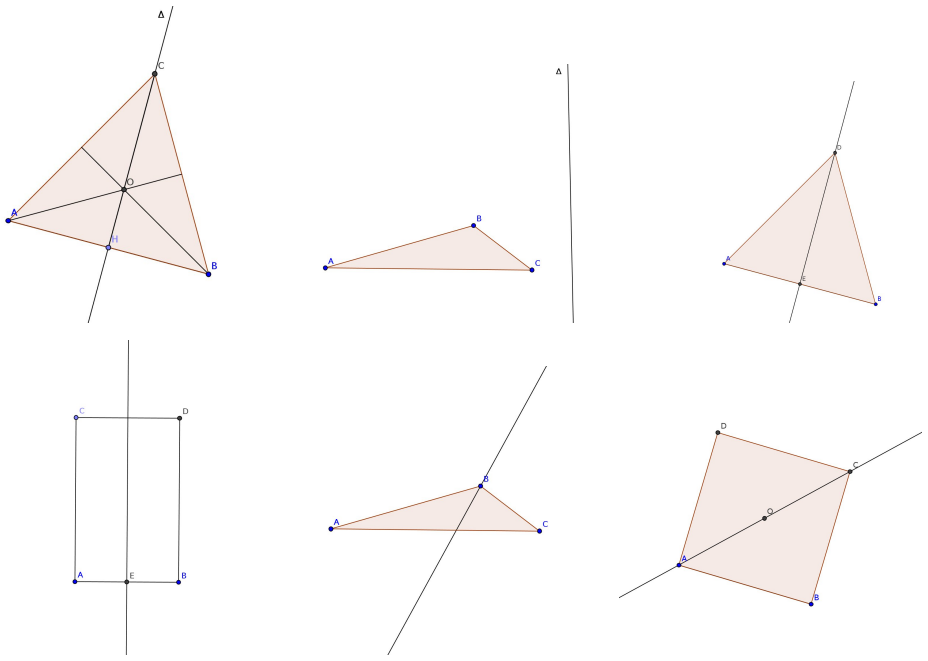
The collaborative task is a team work in a foreign language and members are called below Alice, Bob, Charlie, David and Emmy. Feel free to share roles.

2 Demonstrating geometric functions with the Labosaïque

1. A first generic geometric function : Alice, Bob and Charlie sketch diagrams for the poster following the instructions and answering the questions below.
   (a) Let O be a point and angle $\dfrac{360^\circ}{\mathbf{3}} = 120^\circ$. Consider rotation about center point O by angle 120°. Take a point M and rotate about O by angle 120° to obtain point M'. This function giving point M' starting with point M is the rotation denoted by r_3.
   (b) Choose a shape suggested in the given documents (scalene triangle, equilateral triangle, square) and rotate it under r_3. Is this image shape the same as the starting shape ? Do both shapes coincide exactly on the same spot ? Explain why or why not.
   (c) Can you find other shapes whose image under r_3 coincides exactly with the starting shape ?



2. A second generic geometric function : Bob and Charlie sketch diagrams for the poster following the instructions below.
   (a) Let Δ be a straight line and consider the reflection of axis Δ. *For further reasons this line is taken passing through point O.* Take any point N and reflect it about Δ to obtain a new point N'. This function giving point N' starting with point N is the reflection denoted by s_Δ.
   (b) Choose a shape suggested in the given documents (scalene triangle, equilateral triangle, square) and reflect it under s_Δ. Is this image shape the same as the starting shape ? Do both shapes coincide exactly on the same spot ? Explain why or why not.
   (c) Can you find other shapes whose image under s_Δ coincides exactly with the starting shape ?



3. In the following all team members combine the effects of these two functions r_3 and s_Δ on one point and draw the corresponding diagrams.
   (a) Consider one point M (or shape F), apply reflection s_Δ two times meaning s_Δ applied to M giving point M' and s_Δ applied again to M' giving point M'' (or shape F'').
   (b) Consider the new function starting with point M and obtaining point M'' as an image of M. What is its effect on the starting point ? Why ?
   (c) Repeat this procedure to rotation r_3. What can you observe ?
   (d) Repeat the process for function $s_\Delta r_3$ meaning apply r_3 first to M to get M' and then apply s_Δ to M' to get M'' in this order. This new function starting with point M and obtaining point M'' as an image of M is denoted by $s_\Delta r_3$.
   (e) Repeat the process given in the preceding question for function $r_3 s_\Delta$. Be careful to write letters in the correct order.
   (f) Have functions $s_\Delta r_3$ and $r_3 s_\Delta$ the same effect on any point M ?
4. Alice and Bob demonstrate these geometric functions on shapes using the Labosaïque equipment to allow visitors to understand their findings above. Ask the teachers for help to use the equipment.
5. This is additional information for all.
   (a) Denote by $C\mathbf{3}$ the set of functions r_3, $r_3 r_3$, $r_3 r_3 r_3$. Can you describe these functions in a simple manner ?
   (b) The set of all functions obtained by combining rotation r_3 and reflection s_Δ is denoted by $D\mathbf{3}$.
6. Charlie and David explain to Emmy the role played by the whole number **3** in the previous reasoning leading to sets $C\mathbf{3}$ and $D\mathbf{3}$.
7. What are the functions used for the sets denoted by $D\mathbf{2}$, $D\mathbf{4}$ and $D\mathbf{6}$? For each function state clearly its elements which are center point and angle for the rotation and axis of symmetry for the reflection.
8. Emmy draws diagrams to show these sets denoted by $D\mathbf{2}$, $D\mathbf{4}$ and $D\mathbf{6}$.

3 Globally invariant pattern under a geometric function

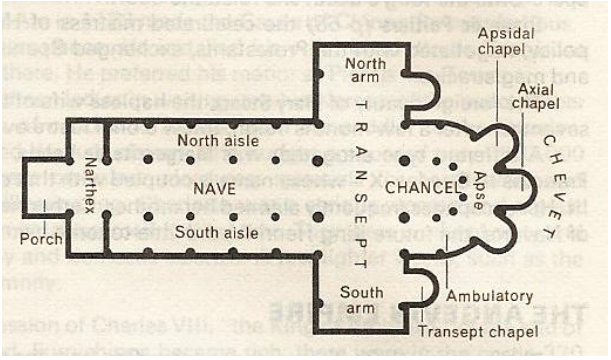
A given shape is called **globally invariant under the geometric function f** if its image under f stays the same.

1. An equilateral triangle is globally invariant under any function in C3 whose center point is also the center of the triangle. Alice draws a diagram to show this property. Bob draws another sort of triangle and experiences the effect of the same function to decided if his triangle stays globally invariant or not.
2. A square is globally invariant under any function in C4 whose center point is also the center of the square. What is C4 ? David draws a diagram to show this property. Charlie draws a rectangle and experiences the effect of the same function to decide if his rectangle stays globally invariant or not.

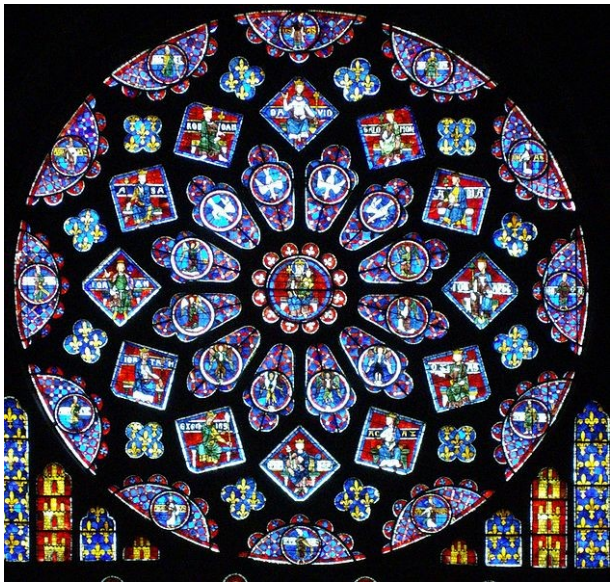
Now, we apply these mathematical ideas to art and architecture.

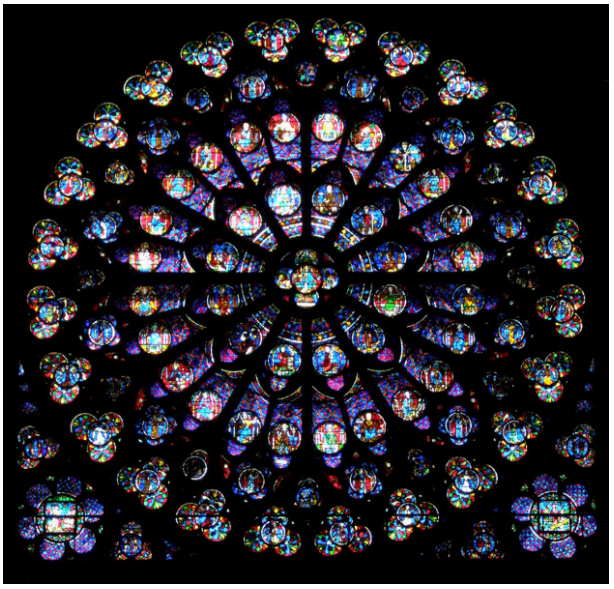
4 Looking for a geometric pattern in architecture

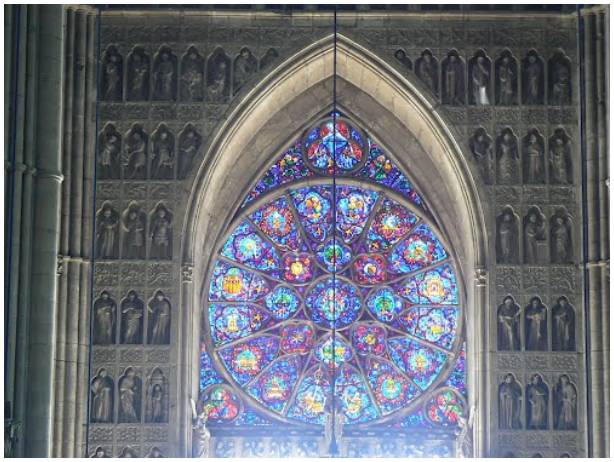
1. Below is the diagram of a church. Each member of the team has to locate on the plan where the picture of a motif has been taken in the church.



2. For each decorative pattern above show under which geometric function it stays globally invariant. Explain.
3. For each function used above draw clearly all its elements on the given photos.
4. Look for other geometric patterns in this architectural site of the Ladies Abbey church and comment on their mathematical insight.
5. Give a mathematical insight on the following rosette shaped windows of cathedrals respectively of Chartres, Paris, Reims and Rouen.







5 Provided material

- The Labosaïque equipment
- a measure tape
- a combined protractor square set ruler (a "Math en Main" present to each participant)
- copies of geometric drawings (scalene triangle, equilateral triangle, square) to be completed
- blank sheets for further drawings to be glued onto the poster
- separate photos of motifs in the church and the church plan