

This scatter plots below show the pregnancy rate of 15-17 year old young women racially classified as Black and Hispanic in an English speaking country. The birth rate is reported as births per 1000 women.

- Describe the situation which is developed in the document.
- Explain from the scatter plots if it appears that efforts to educate youth about abstinence and safe sex are having an impact.
- A scientist suggests two models, the cubic function model for each of the data sets is given below.

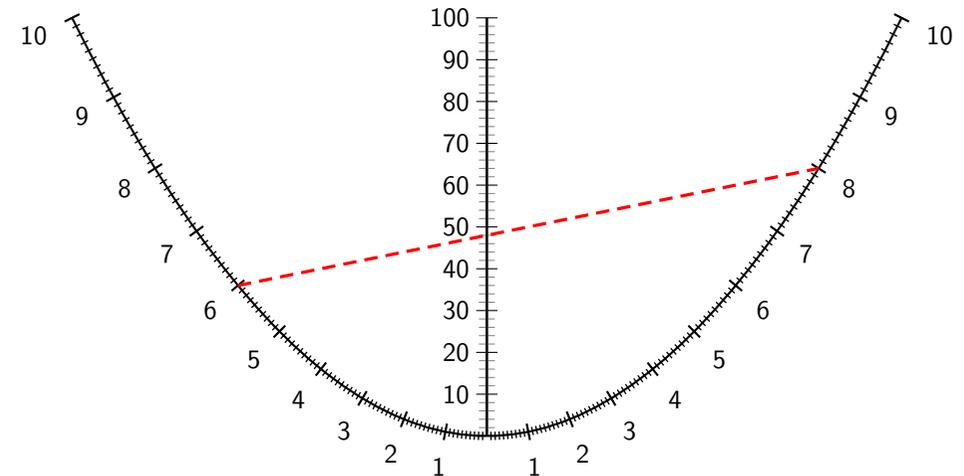
**Hispanic birth rate:**  $h(x) = 0.0280x^3 - 0.718x^2 + 3.34x + 65.9$  ;  $r^2 = 0.997$

**Black birth rate:**  $b(x) = 0.0215x^3 - 0.475x^2 - 0.846x + 82.3$  ;  $r^2 = 0.999$ .

Enter the two functions in your graph calculator and comment on the reliability of the models.

A *nomogram*, also called an abacus, is a graphical calculating device designed to allow the approximate graphical computation of a function.

The following nomogram is based on the graph of a parabola. Choose point 6 on one half of the parabola and point 8 on the other half of the parabola, then draw the line and note the intersection point with the vertical axis: that's 48 ... which is  $6 \times 8$ !



### Tasks

- Choose two integers and verify that their product can be found on the above graph.
- Consider the parabola  $y = x^2$ . State the properties of this elementary function and the properties of its graph.
- Let  $a$  and  $b$  be two distinct real numbers. The goal here is to explain why this method works.
  - Compute the slope of the line that crosses the parabola  $y = x^2$  where  $x = a$  and where  $x = b$  on the parabola.
  - Deduce the equation of the line and conclude.
- Explain how to compute  $60/7$  approximately using this nomogram.
- Let's consider two integers  $a$  and  $b$  greater than or equal to 2 on both sides of the parabola, join them and look closely at the intersection point with the vertical axis.
  - Explain why 3 and 5 will not be reached.
  - Give some examples of other integers that will not be reached.
  - Give the major arithmetical property that all these integers share.

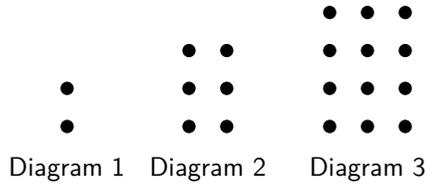
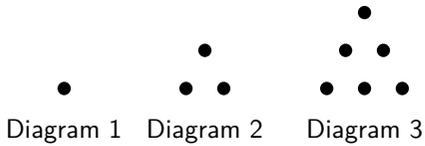
**BACCALAURÉAT - DNL Mathématiques/Anglais - Session 2012**  
**How can numbers have a geometric shape? – Sujet 3**

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Although numbers do not have a geometric shape, some can be represented by dots that can be put into a geometric shape. Here are some examples.

**Triangular numbers:**

**Oblong numbers:**



**Tasks**

1. Make a sketch of diagram 4 in each sequence.  
 Explain how you made them.
2. Complete the empty squares in the table for diagrams 4 and 5.

Diagram $n$	1	2	3	4	5	$n$
Triangular number: number of dots	1	3	6			$T_n$
Oblong number: number of dots	2	6	12			$O_n$

3. Using the table above, say what conclusion you can draw.
4. Explain: " $T_n$  is the sum of the  $n$  first whole numbers".
5. Can you conjecture an equivalent property of oblong numbers?
6. Prove the following equalities:

$$T_n = \frac{n(n+1)}{2} \quad O_n = n(n+1)$$

7. The expressions of  $T_n$  and  $O_n$  confirm what is found in questions 3 and 5. Discuss.

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**The Cantor set – Sujet 4**

*(This is a conversation between Henry and Anne-Lida.)*

**Henry:** Georg Cantor was a German mathematician who invented a very curious set in about 1883 [...]. To get a Cantor set you start with a line segment of length 1, and remove its middle third. Now remove the middle third of each remaining piece.

Repeat, forever. What is left is the Cantor set. (Figure below)

**Anne-Lida :** I don't see how there can be anything left, Henry.

**Henry:** Oh, but there is. All the end-points of all the smaller segments are left, for a start. And many others. But you are right in one way, my dear. What is the length of the Cantor set ?

**Anne-Lida :** Its ends are distance 1 apart, Henry.

**Henry:** No, I meant the length not counting the gaps.

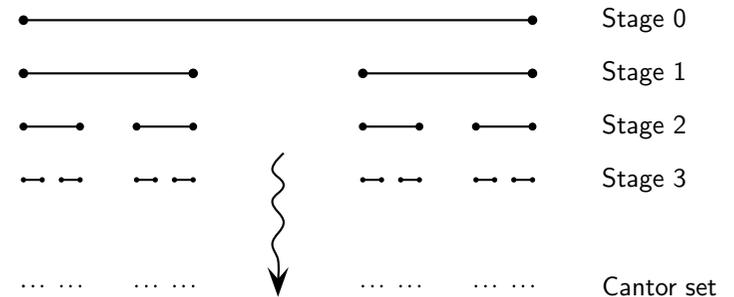
**Anne-Lida :** I have no idea, Henry. But it looks very small to me. The set is mostly holes.

...

**Henry:** The length reduces to  $\frac{2}{3}$  the size at each stage, so the total length after the  $n$ th stage is  $\left(\frac{2}{3}\right)^n$ . As  $n$  tends to infinity, this tends to 0. The length of the Cantor set is zero.

Ian Stewart, *Game, Set & Math, enigmas and conundrums*,  
 Dover publications, 2007.

Stage	Segment length	Number of segments	Total set length	Number of endpoints
0	1	1	1	2
1	$\frac{1}{3}$	2	$2 \times \frac{1}{3} = \frac{2}{3}$	2
2				
3				
4				



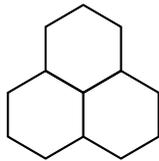
*Construction of the Cantor set by removing middle thirds.  
 Its length is 0; but contains infinitely many points.*

**Tasks**

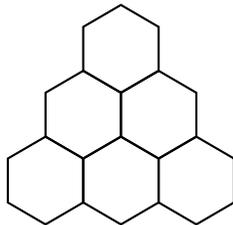
1. Fill in the table above with integers or fractions raised to the appropriate power.
2. Conjecture the line of the table at stage  $n$ .
3. Explain why the Cantor set at stage  $n$  contains more than  $2^{n+1}$  points.
4. Explain why the Cantor set at stage  $n$  has length  $\left(\frac{2}{3}\right)^n$ .
5. Let  $n$  go to infinity and give the length of the Cantor set. Justify your answer.
6. How many points are contained in this Cantor set?
7. Using the preceding results, do you think it is possible for a set to be simultaneously of length zero and non empty ? Explain your reasoning.



Step 1



Step 2



Step 3

Jeremy a bee-keeper is doing an investigation which involves making patterns of hexagons with sticks. The first of these three patterns are shown above.

### Tasks

- Describe the easiest way to find the extra number of hexagons and the extra number of sticks necessary in step 4.
- Copy and complete this table

number of row, $n$	1	2	3	4
number of hexagons, $h_n$	1	3		
number of sticks, $s_n$	6			

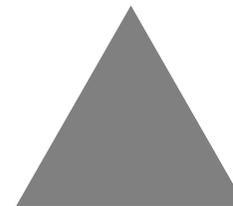
- Jeremy doesn't know which formula to use to calculate  $s_n$  in terms of  $n$ .

$$s_n = 2n^2 + 4n \quad s_n = \frac{3}{2}n^2 + \frac{9}{2}n \quad s_n = \frac{5}{2}n^2 + \frac{3}{2}n + 2$$

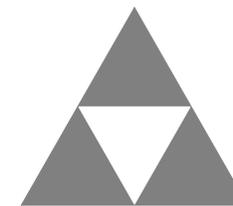
- Help him to choose the right formula.
  - Explain the reasons why you have arrived at this result.
- Find the number of rows for the number of sticks to be 105.

Fractals are geometric shapes that are self-similar at different scale. They are formed by applying the same procedure over and over again. Sierpinski's triangle is one of the most famous examples of a fractal, named after the Polish mathematician Waclaw Sierpinski (1882-1969) who described it in 1915.

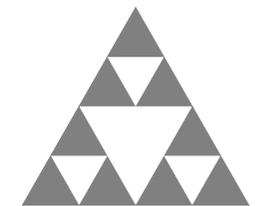
Sierpinski's triangle is created by infinitely repeating the construction process shown below. In step 0, an equilateral triangle is drawn.



Step 0



Step 1



Step 2

The goal is to look at the pattern and to find relations between the step number and the different geometric elements.

### Tasks

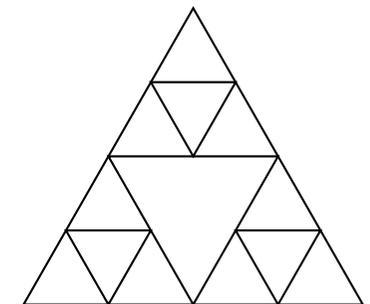
- Describe the procedure to construct the next step. (You could use the adjoining figure.)

- Number of remaining triangles (grey ones)**

In step 0, there is one triangle remaining.

How many triangles remain in step 1, step 2?

Explain the pattern



- Area of Sierpinski's triangle (grey area)**

Let the area of the original triangle be 1 (triangular unit).

In step 1, justify that the area of Sierpinski's triangle is  $3/4$ .

What is the area of the Sierpinski's triangle in step 2?

Deduce the formula for the area of the Sierpinski's triangle for any step.

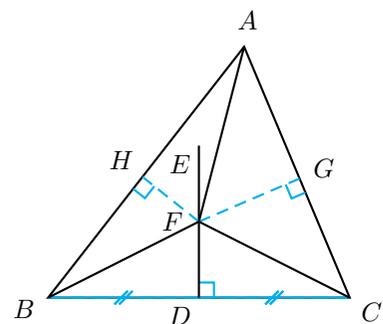
- What can you say about the number of triangles and the area of Sierpinski's triangle by repeating the process infinitely?

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All triangles are isosceles... – Sujet 8

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The English author Lewis Carroll (1832 –1898) found fame after the publication of his book *Alice's Adventures in Wonderland*. But less well known, is the “other” Lewis Carroll, the mathematician at Christ Church College, Oxford. Within the academic discipline of mathematics, he worked primarily in the fields of recreational mathematics, producing a dozen books. In one of these, he developed the following proof that every triangle has two equal sides.

Let  $ABC$  be any triangle. Bisect  $BC$  at  $D$  and from  $D$  draw  $DE$  at right angles to  $BC$ . Draw line  $AF$  that bisects  $\angle BAC$ . Join  $FB$ ,  $FC$ , and from  $F$ , draw  $FG$ ,  $FH$ , at right angles to  $AC$ ,  $AB$ .



1.  $\angle FGA = \angle FHA$ ;
2.  $\angle FAG = \angle FAH$ ;
3. Triangles  $AFG$  and  $AFH$  have the side  $AF$  in common;
4. Therefore,  $AG = AH$ .
5. Similarly, we have  $FG = FH$ .
6.  $FB = FC$ .
7. Therefore, combining steps (5) and (6), we obtain  $GC = HB$ .
8. Combining steps (4) and (7),  $AC = AB$ .

Therefore, triangle  $ABC$  is isosceles.

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Adapted from *Rediscovered Lewis Carrol puzzles*, by Edward Wakeling.

## Tasks

1. Propose a short summary of the subject and give your first impression.
2. Line  $DE$  is known as a remarkable line in triangle  $ABC$ : give some more information about such a line.
3. Line  $AF$  is known as a remarkable line in triangle  $ABC$ : give some more information about such a line.
4. Resume step by step by step Carroll's proof and justify each statement.
5. By drawing your own picture accurately, find out the fallacy in Carroll's proof.