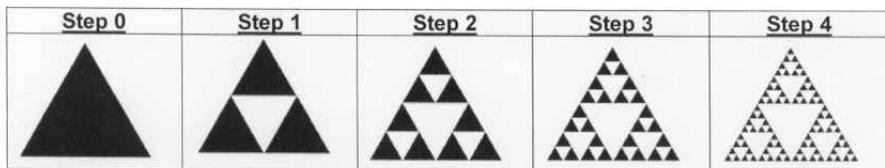


The Sierpinski Triangle

Don't spend more than 10 minutes preparing question 1 and watch the video.

How to draw a Sierpinski triangle?

- Draw an equilateral triangle.
- Connect the midpoints of each side. Cut out the triangle in the centre.
- You are now left with three equilateral triangles. Connect the midpoints of each side of these triangles and cut out the triangle in the centre.
- Follow the same procedure again and again. You obtain a Sierpinski triangle.



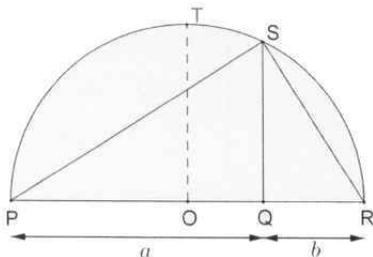
- Describe the construction in your own words, focusing on the number of white triangles that you remove at each step.
 - Simon says: "I have built a 10- step Sierpinski triangle. I had to cut more than 20,000 triangles". Explain whether this is true or false.
- Watch the video "*The chaos game*".
 - Explain how to play the chaos game. Draw an example on the board.
 - Say in what way the chaos game and the Sierpinski triangle are related.

Remember that

$$1 + q + q^2 + \dots + q^n = \frac{1 - q^{n+1}}{1 - q}$$

The geometric mean : just another type of average

Imagine you own a rectangular plot of land and you want to build on it a square-shaped house. Your plot, however, is rather long and narrow (let a and b be the lengths of its sides), so it doesn't provide enough space for your dream house. You therefore propose your realtor* to trade it off for a square-shaped plot of equal area. What should be the side of this square ? This value is called the geometric mean of a and b . (...)



The figure on the left depicts the geometric mean the way the Greeks looked at it (in a geometric way). We think of a and b as the lengths of two line segments, and we draw them end to end along a straight line. We then draw a semi-circle with this line as diameter and erect a perpendicular to it at the point where a and b are joined. The height of this perpendicular is $G = \sqrt{ab}$.

Adapted from *Beautiful geometry*, by Eli Maor and Eugen Jost
Princeton University Press, Princeton and Oxford.

*Glossary : realtor = agent immobilier

Tasks :

1. In your own words, explain the situation and answer the question at the end of the first paragraph.
2. Choose a couple of examples and compare $G = \sqrt{ab}$, the geometric mean of two positive numbers a and b , to their arithmetic (ordinary) mean $M = \frac{a+b}{2}$.

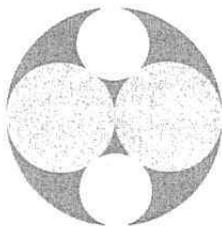
Is your observation true for all values of a and b ? (You may want to compare M^2 and G^2 in the general case.)

3. Explain the connection between the figure and the second paragraph of the text. Spot all the right angles in the figure and prove that $SQ = \sqrt{ab}$.

Explain the geometric meaning of $a+b$ and $\frac{a+b}{2}$.

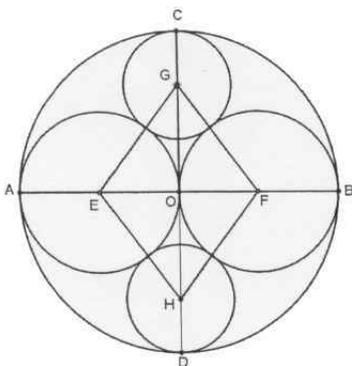
4. Give a geometric proof that G is always smaller than or equal to M .

This is the logo of a computer science company:

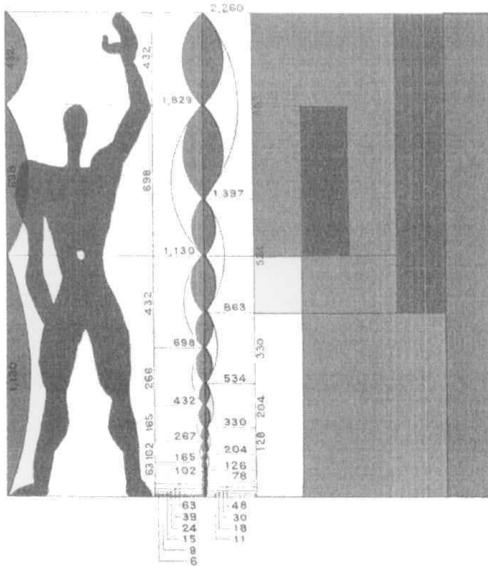


- The largest circle has radius $R = 6$ cm.
- The smallest circles have radii r in cm.
- The circles are tangents.

The following figure shows the logo in mathematical details:



- 1) Describe the figure using mathematical words.
 - 2) Find the radius of the intermediate circles.
 - 3) Show that the radii of the smallest circles are $r = 2$ cm.
 - 4) Compute the side lengths of the triangle OFG. Comment on your results.
 - 5) Choose one of the following tasks and then execute it:
 - a) Compute the fraction of the largest circle occupied by the four inner circles.
- or
- b) Compute the area of the rhombus EGFH



One of the strongest advocates for the application of the Golden Ratio to art and architecture was the famous Swiss-French architect and Painter Le Corbusier (Charles-Edouard Jeanneret, 1887- 1965).

The Modulor (see picture) was supposed to provide « a harmonic measure to the human scale, universally applicable to architecture and mechanics. » In the spirit of Vitruvian man and the general philosophical commitment to discover a proportion system equivalent to that of natural creation, the

Modulor was based on human proportions. A six-foot (about 183-centimeter) man, somewhat resembling the familiar logo of the “Michelin man,” with his arm upraised (to a height of 226 cm; 7’5”), was inserted into a square . The ratio of the height of the man (183 cm; 6’) to the height of his navel* (at the mid-point of 113 cm; 3’8.5”) was taken precisely in a Golden Ratio.

adapted from Golden Ratio in Art & Architecture Nikhat Parveen, UGA

*Navel: *nombril*

Tasks:

1. Describe the situation presented in the document.
2. The Golden Ratio is the positive solution to $x^2 - x - 1 = 0$, calculate its exact value and check that 1.618 is the approximation to 3 decimal places with your calculator.
3. The numbers 6, 9, 15, 24, 39..., 267 and 432 constitute a sequence. What should the next term be? Compare with the values of the document.
4. Calculate $9/6$, $15/9$,... Check the following quote from the text: ‘The ratio of the height of the man (183 cm; 6’) to the height of his navel (at the mid-point of 113 cm; 3’8.5”) was taken precisely in a Golden Ratio’.
5. The Golden Ratio is supposed to provide a harmonic proportion. Using your knowledge, give examples you have heard about in different fields such as nature, architecture, maths...



Doubling my money -

Do you love money? Henry does.

He likes to put his money to work when he is not using it. He invests it.

He puts \$1,000 into an investment that will earn him 4% interest, compounded annually. Hence his investment will earn \$40 after one year.

After one year his investment will be worth \$1,040 and he will earn 4% interest on this new amount.

1. Assuming his investment continues to grow at 4%, calculate how much will it be worth after two years, after three years.

Henry got into a heated discussion with his friend John over how long it will take for him to double his money with this investment. Henry says it will take 25 years, because 25 years of 4% increases will add up to a 100% increase. John says it will be only 18 years.

2. Determine who is right and then explain your calculation to the jury.
3. Watch the video : "rule of 72" and explain when and how to use the rule of 72.

Apply the rule to a 9% rate.

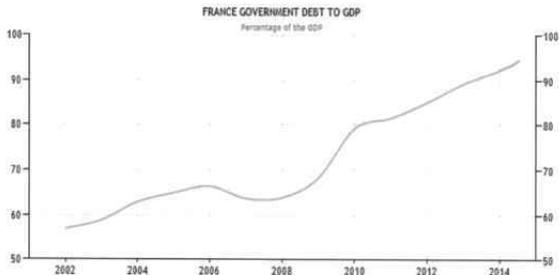
4. John wants to find an investment which will double his money in 8 years. Explain how to determine such an interest rate with or without the previous rule.

French debt tops symbolic level of 2 trillion euros

Adapted from Deutsche Welle 09/30/2014:

"France's public debt has shot up to just over 2 trillion euros for the first time, according to official figures for the second quarter (end of June), reaching 95.1 percent of *GDP, more than 35 percent higher than the EU's fiscal rules allow.

National Statistics Agency INSEE said that France's national debt reached 2.023 trillion euros, an increase of 1.1 percent compared to the previous quarter."



Source : www.tradingeconomics.com

*GDP (Gross Domestic Product) is an indicator used to measure the economic performance of a whole country.

Tasks

1. Present the situation by making clear connections between the text and the graph.
2. France's national debt reached 2.023 trillion euros at the end of June 2014.
 - a) Given that it represented 95.1 percent of GDP, compute the GDP at the end of June 2014.
 - b) Given that it represented an increase of 1.1 percent compared to the previous quarter, compute the value of France's debt at the end of March 2014.
3. The graph, which presents the evolution of the debt as a percentage of the GDP since the year 2002, leads us to define a linear regression. The following table defines two variables where the index $x = 1$ corresponds to year 2002 and the debt y given in % of GDP :

| Year | 2002 | 2004 | 2006 | 2008 | 2010 | 2012 | 2014 |
|-----------|------|------|------|------|------|------|------|
| Index x | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| Debt y | 56.9 | 62.9 | 66.4 | 63.9 | 79.2 | 85 | 95.1 |

- a) Explain the concept of *linear regression*.

To state the equation of the regression line, we will use the method known as the **Mayer Line Method**.

- b) Divide the data into two groups: group 1 with the first three points, and group 2 with the four other points, and for each group, calculate the mean of the x values and the mean of the y values rounded to 1 decimal place.
- c) From this, create two new points named G_1 and G_2 , called the "mean points", and compute the equation of line (G_1G_2) called the "Mayer Line".
- d) According to this model, what would France's national debt be in 2020?

1089 and all that.

TASKS :

1. Watch the video « 1089 trick explained ».
<http://www.youtube.com/watch?v=ec0xnIywEqk>
2. Perform this magic trick for the jury using numbers : 763 then 268 and finally 525.
3. Explain if this 1089 trick works for all three digit numbers.
4. Ask the jury for another number and perform the trick again.
5. Give a *proof* of the trick.
6. If time, choose one of the following tasks and then execute it.
 - a) Explain another trick you know on numbers.

Or

- b) Listen carefully to the script of the video and pick all words related to mathematics. Explain these words.



Watch the video of James Grime who is a British mathematician from the University of Cambridge.

- 1) Explain the link between the video and the number 23.
- 2) Explain what kind of mathematical problem James Grime wants to solve and the manner he will proceed.
- 3) Towards the resolution: Consider each member of a 3-people group, one by one.

The first person will have his/her birthday on a particular day.

- a) What is the probability of the second person having a different birthday from the first?
- b) Explain why the probability of the third person having a birthday different from the first and second is $\frac{364 \times 363}{365^2}$.
- c) Deduce the probability that at least two of these three people have the same birthday.

As expected, it is not likely that any 2 out of 3 people will have the same birthday.

- 4) State the calculation that helps you solve James Grime's problem.
If you can't find the result with your calculator, the jury will give it to you so that you can justify the choice of the number 23.