

Document 1

Escherichia Coli (E. Coli) is a common type of **bacteria** that infects food and water. It is commonly found in the human lower intestine and helps to digest food. It takes only 20 minutes to one E. Coli cell to reproduce in two cells.

Most E. Coli strains are harmless, but few can cause serious illness, especially in children. Its significance as a public health problem was recognised in 1932, following an outbreak in the United States of America.

Adapted from *Wikipedia* and *World Health Organisation*

Glossary: Strain: *souche*

Document 2

In August 2014, Lake Hiawatha Beach in Minnesota, USA, was closed because of E. Coli.

Lake Hiawatha
Surface area: 217,000 m ²
Depth: 10 m

Single sample maximum allowable density par 1m³ for E. Coli: 2,350,000.

Tasks

- Calculate the number of E. Coli obtained from one cell after 20 minutes, and longer, up to 2 hours. Show results in a table:

Time (mn)	0	20	40	60	80	100	120
Number of E. Coli							

- Sketch a diagram to explain E. Coli population growth.
- Conjecture a formula to describe this number of E. Coli after n times 20 minutes.
- Assuming that Lake Hiawatha is a cuboid, explain how you can calculate its volume and how long would it take to one E. Coli cell to contaminate the lake.

On May 25, 2008, the average price for unleaded gasoline in Florida was \$4 per gallon (source: www.floridastategasprices.com)

A driver in Tallahassee, Florida, plans a trip. The driver owns a SUV that gets 18 miles per gallon. Dollar Rent A Car, Inc., offers a Ford Focus car rental in Tallahassee for \$40 administrative fees included. It is estimated that car gets 35 miles per gallon on the highway.

You want to know when it will be in the driver's interest to rent the Ford Focus instead of driving his own SUV.

Adapted from *MIRL Activity library, volume I* by Frank C. Wilson

Glossary: SUV: Sport Utility Vehicle

Tasks

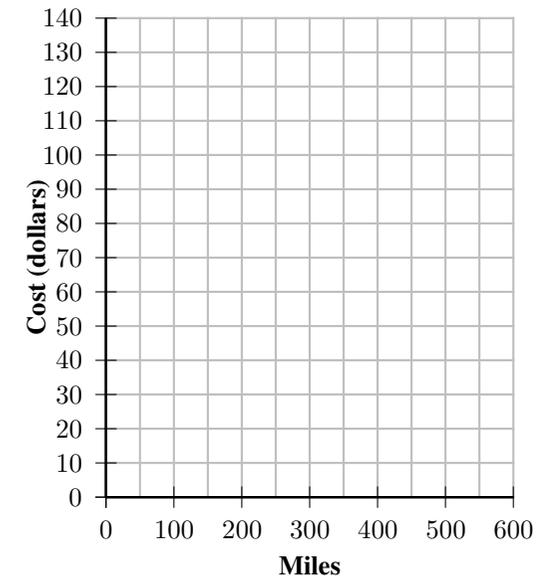
- Explain which of the 2 cars has the lowest gasoline consumption.
- Complete the tables and sketch a graph that may help answer the problem.

For the SUV

Miles	0	18	180
Cost			

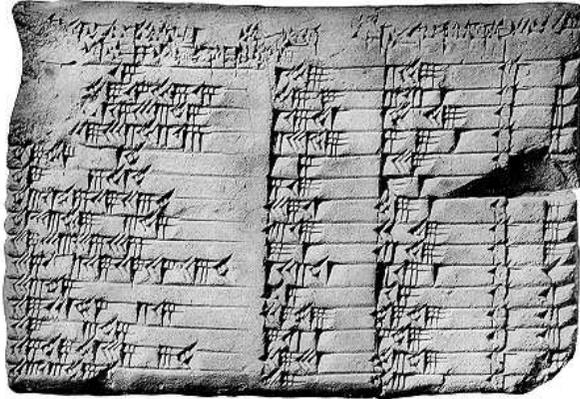
For the Ford Focus

Miles	0	35	350
Cost			



- Propose another way of solving the problem
- The driver plans to drive to Orlando, Florida, a distance of a roughly 515 miles round trip. Advise the driver about renting the Ford Focus or not.

A Pythagorean triple is a triple of whole numbers (a, b, c) where $a^2 + b^2 = c^2$. Below is an ancient Babylonian tablet listing fifteen Pythagorean triples. It is called Plimpton 322 (George Arthur Plimpton donated it to Columbia University).



Tasks

1. Justify the title “*Pythagorean triples*”. Give an example.
2. Here is a list of triples
 - (a) $(5, 12, 13)$
 - (b) $(9, 10, 13)$
 - (c) $(8, 15, 17)$
 - (d) $(4, 12, 4\sqrt{10})$
 - (e) $(30, 40, 50)$
 - (i) Justify which of the triples above are **not** Pythagorean triples.
 - (ii) Explain why the set of Pythagorean triples is or is not endless.
3. A *twin* Pythagorean triple is a Pythagorean triple for which two values are consecutive numbers. Look at $(5, 12, 13)$ and $(7, 24, 25)$. In both triples, the largest numbers are consecutive numbers.
 - (i) Let m and n be the two first numbers. Write the relation with m and n so that $(m, n, n + 1)$ is such a twin Pythagorean triple, and prove that m is necessarily odd (*Hint: for any whole number a , if a^2 is odd then a is odd.*)
 - (ii) Ask the jury for an odd number, and using the previous relation, compute the twin triple with this odd number as the smallest number.
4. Find out all the Pythagorean triples with three consecutive numbers.

Tasks

Watch and listen to the video “*Why sixty*” from 0 to 2’05

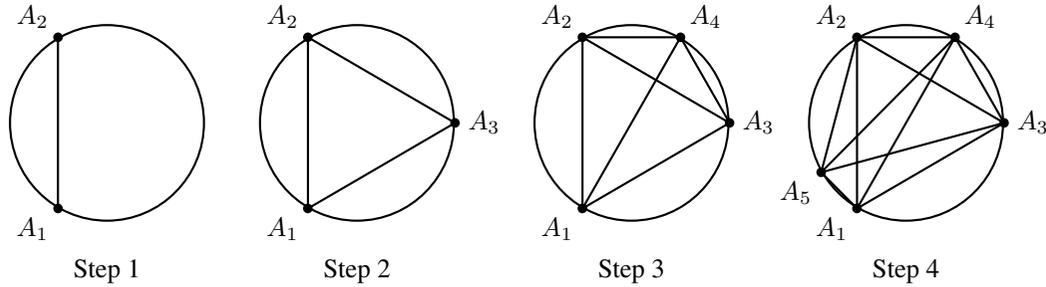
<https://www.youtube.com/watch?v=R9m2jck1f90>

1. From the video explain why number 60 was interesting for Babylonians.
2. Show the jury how to count to number 43 on your hands in the Babylonian way.
3. Indicate what number is shown on the picture below.



4. Nowadays numbers are given in base 10.
For example, 5407 means $5 \times 10^3 + 4 \times 10^2 + 0 \times 10^1 + 7 \times 10^0$.
 - (a) Explain why counting to 43 with your hands in the Babylonian way is equivalent to counting in base 12.
 - (b) Write 2 016 in base 12.
5. Find out remaining signs of base 12 in our everyday life.

In the sequence of circle diagrams below, each point is joined to every other point on the circle by a straight line. Here $A_1A_2A_3$ is an equilateral triangle with $A_1A_2 = 1$.

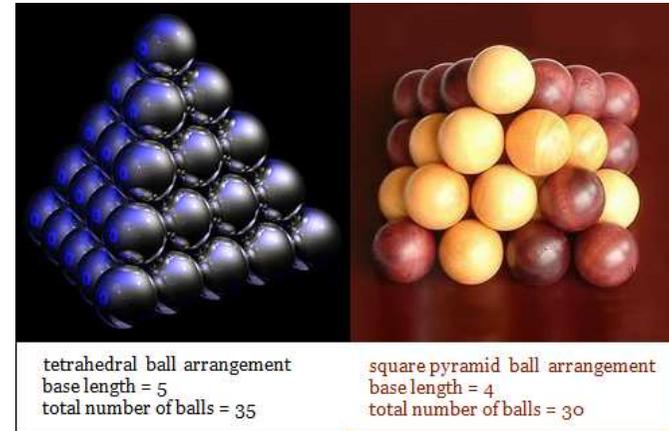


Tasks

- Perpendicular bisectors have been used to construct this sequence of points. Explain how and give a possible location for A_6 .
- Define a formula connecting the number of line ℓ and the number of points p .
- Assuming $A_2A_4 = \frac{\sqrt{3}}{3}$, calculate the area of $A_1A_2A_3A_4$ specifying the steps of the calculation.

Adapted from *Higher GCSE Mathematics*, D. Rayner, ISBN:978-019-914-5744.

There are two ways to stack spheres into a pyramid. One (tetrahedron ball arrangement) is to start with a triangular base and build up the layers with successively smaller triangles. Another (square pyramid ball arrangement) is to start with a square base, adding layers of smaller squares on top until you reach the apex (see image below). Mathematically, both of these arrangements are the most efficient way to pack spheres in three dimensional space



Imagine that you were on a sailing ship a few hundred years ago and, in order to protect yourself from pirates, you had two cannons. Cannons need cannon balls and it is well known that the best way to stack cannon balls is to arrange them as pyramids.

- Stacking 10 cannon balls into a pyramid. Explain the idea.
- You decide to list all the numbers of cannon balls that can be stacked into a pyramid. The mathematical genius of the ship says that he has already done it and gives you the table below

Base length	1	2	3	4	5	6	7
Total number of balls in the base triangle	1	3	6	10	15		
Total number of balls in the tetrahedron pyramid	1	4	10	20	35		
Total number of balls in the square pyramid	1	5	14	30	55		

Complete the last two columns and explain your method

- The Captain of the ship has a maximum of 100 cannon balls to stack. Stack these balls into two pyramids the best as possible. Explain.

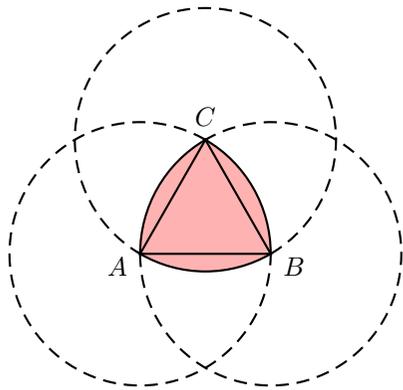
BACCALAURÉAT - DNL Mathématiques/Anglais - Session 2016
The Reuleaux triangle – Sujet 8

Franz Reuleaux (1829-1905) was a German scientist and engineer who is regarded as the founder of modern kinematics^a and machine design. His name is mainly remembered today for an idea that would lead to some unexpected practical uses.

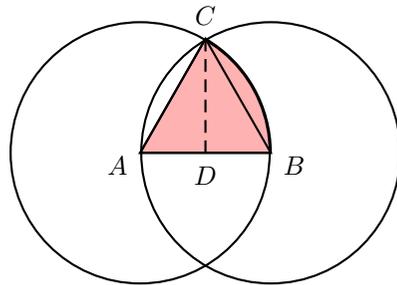
With a compass, draw three identical circles, each of radius r and each passing through the centre of the other two. The overlapping area is called the *Reuleaux triangle* (the “triangle” made by the circular arcs connecting A , B and C in picture 1).

Adapted from *Beautiful geometry*, by Eli Maor and Eugen Jost.
 Princeton University Press, Princeton and Oxford.

^aKinematics is the part of mechanics which describes movement



Picture 1



Picture 2

To facilitate pronunciation, refer to the Reuleaux triangle as the “*R triangle*”.

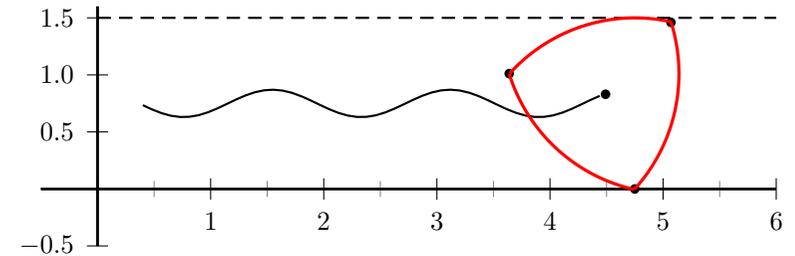
Tasks

1. Describe the (ordinary) triangle ABC (sides, angles).
 Suggest a point O that could be called the centre of the Reuleaux triangle and explain how to construct this point.
2. Calculate the perimeter of the Reuleaux triangle in terms of r .
3. (a) Describe the distance between a vertex of the Reuleaux triangle and any point on the opposite “side”.

- (b) Watch the video provided and explain how the Reuleaux triangle could be used as a “wheel” in contact with two parallel lines (picture below).

<https://www.youtube.com/watch?v=Bvn4nmM0qyY>

- (c) Explain the meaning of the squiggly line in the middle.



- (d) Calculate, in terms of r , the areas of the shaded circular sector in Picture 2, of the ordinary triangle ABC , and finally of the Reuleaux triangle.

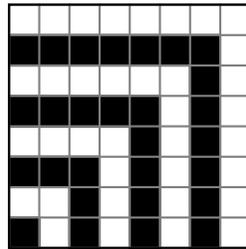
BACCALAURÉAT - DNL Mathématiques/Anglais - Session 2016
Proofs without words – Sujet 4

In mathematics, a proof without words is a proof of an identity or mathematical statement which can be presented as self-evident by a diagram without any accompanying explanatory text. When the diagram shows a particular case of a general statement, to be a proof, it must be generalisable.

A proof without words for the sum of odd numbers theorem

$$1 + 3 + 5 + 7 + \dots + (2n - 1) = n^2$$

The statement that the sum of all positive odd numbers up to $2n - 1$ is a perfect square number, more precisely n^2 here, can be proved without words, using the diagram on the right. The first square consists in one block; so 1 is the first square number. The next strip, composed of white blocks, shows how adding three more blocks makes another square number: 4. The next strip, composed of black blocks, shows how adding five more blocks makes the next square number. This process can be continued indefinitely.



A proof without words for the sum of odd numbers theorem

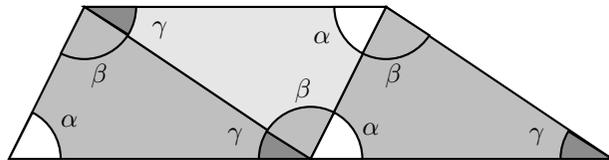
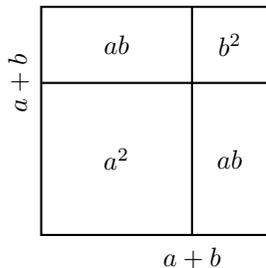
Glossary: beads: *perles*

A proof without words for the area of a circle is given in the video.

<https://www.youtube.com/watch?v=whYqhpc6S6g>

Tasks

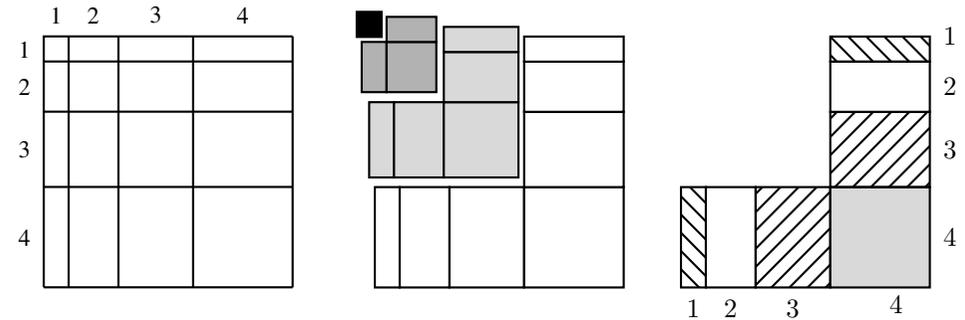
1. Present to the jury the proof without words of the sum odd numbers theorem.
2. Perform and explain to the jury the proof of the formula for the circle's area shown in the video.
3. Define the mathematical statements shown by the diagrams below.



4. Using the diagrams below, prove to the jury the formula

$$(1 + 2 + 3 + \dots + n)^2 = 1^3 + 2^3 + 3^3 + \dots + n^3$$

Hint: explain why this "right-angled elbow" of the third diagram has total area equal to 4^3 .



Sources: Adapted from *Wikipedia*
https://en.wikipedia.org/wiki/Proof_without_words
La géométrie de la chambre à air, J-P Boudine, éditions Vuibert
Des images pour les nombres, Kerléguer et Dumont, éditions du Kangourou