Baccalauréat général – Session 2016
Section européenne, DNL Mathématiques – Anglais
24/05/2016 - Sujet n°1

Académie de Caen

Escherichia Coli

Document 1

Escherichia Coli (E. Coli) is a common type of bacteria that infects food and water. It is commonly found in the human lower intestine and helps to digest food. It takes only 20 minutes to one E. Coli cell to reproduce in two cells.

Most *E. coli* strains are harmless, but a few can cause serious illnesses, especially in children. Its significance as a public health problem was recognized in 1982, following an outbreak in the United States of America.

Adapted : Wikipedia and World Health Organisation

GLOSSARY :Strain : Souche

Document 2

In August 2014, Lake Hiawatha Beach in Minnesota, USA, was closed because of E. Coli.

Lake Hiawatha

Surface area : 217,000 m^2

Depth : 10 m

Single Sample Maximum Allowable Density per $1m^3$ for E coli : 2,350,000

Tasks :

1) Calculate the number of E. Coli obtained from one cell after 20 minutes, and longer, up to 2 hours. Show results in a table :

Time (mn)	0	20	40	60	80	100	120
Number of E Coli							

2) Sketch a diagram to explain the E. Coli population growth.

3) Conjecture a formula to describe this number of E. Coli after n times 20 minutes.

4) Assuming that Lake Hiawatha is a cuboid, explain how you can calculate its volume and how long would it take to one E. Coli cell to contaminate the lake ?

ANSWERS

1) Since it doubles every 20 minutes, it would be 2 after 20 minutes, 4 after 40 minutes, 8 after one hour and then 64 after 2 hours.

2) It is an exponential growth.

3) $C_n=2^n$, it's a geometric sequence with 2 as a common ratio.

4) The volume of the lake is $217{,}000 \times 10=2{,}170{,}000$ m^3. It would need $2{,}170{,}000\times 2{,}350{,}000=50995.10^8$ E.Coli to contaminate the lake.

   Soling $2^n>50995.10^8$ gives $n>42$. Then it would take 43 periods of 20 minutes to contaminate the lake i.e. 14 hours and 20 minutes

Baccalauréat général – Session 2016 **Académie de Caen**
Section européenne, DNL Mathématiques – Anglais
24/05/2016 – SUJET N°2

Travel options - Florida

On May 25, 2008, the average price for unleaded gasoline in Florida was \$4 per gallon (Source: www.floridastategasprices.com).
A driver in Tallahassee, Florida, plans a trip.
The driver owns an SUV that gets 18 miles per gallon.
Dollar Rent A Car, Inc., offers a Ford Focus car rental in Tallahassee for \$40 administrative fees included. It is estimated the car gets 35 miles per gallon on the highway.

You want to know when it will be in the driver's interest to rent the Ford Focus instead of driving his own SUV.

SUV : Sport Utility Vehicle

Extracted from "MIRL Activity library, volume I" by Frank C. Wilson.

Tasks

1) Explain which of the 2 cars has the lowest gasoline consumption.

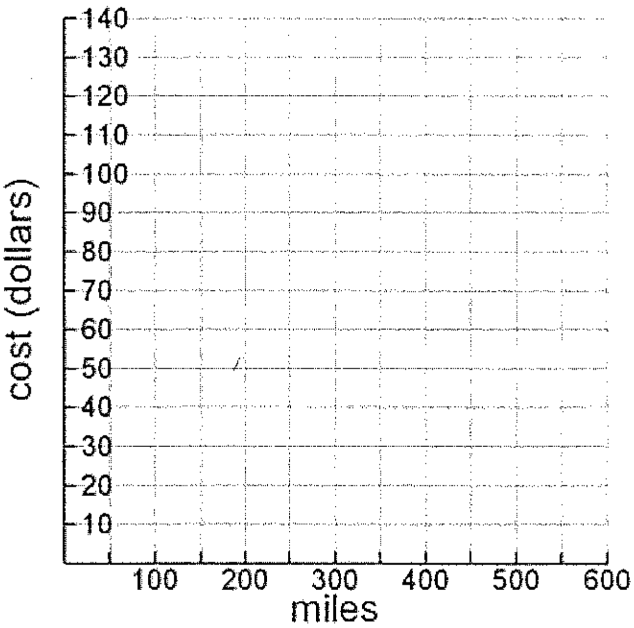
2) Complete the tables and sketch a graph that may help answer the problem.

For the SUV :

miles	0	18	180
cost			

For the Ford Focus :

miles	0	35	350
cost			



3) Propose another way of solving the problem.

4) The driver plans to drive to Orlando, Florida, a distance of a roughly 515 miles round trip. Advise the driver about renting the Ford Focus or not.

ANSWERS

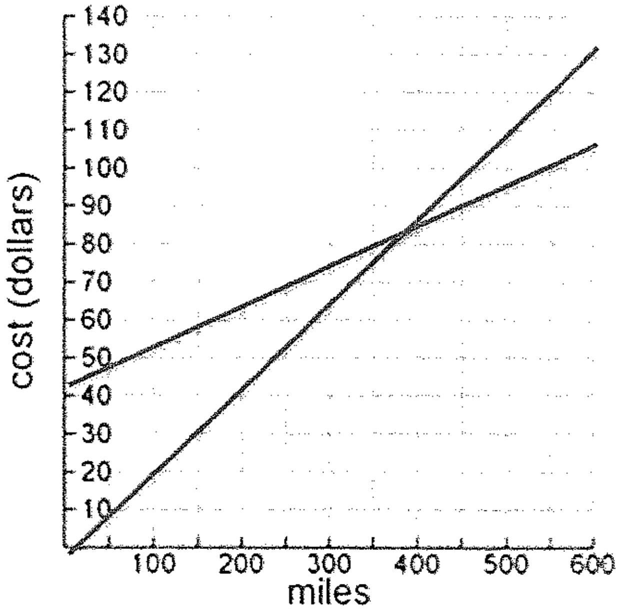
1. Since 35>18 the Ford Focus has the lowest gasoline consumption.
2.

For the SUV :

miles	0	18	180
cost	0	4	40

For the Ford Focus :

miles	0	35	350
cost	40	44	80



It seems to be the driver's interest to rent the Focus for more than 370 miles.

3. Considering x is the number of miles, the driver is better with his own car if $\frac{4x}{18} < 40 + \frac{4x}{35}$ i.e. $\frac{68x}{630} < 40$ i.e. x<370.58 miles.

4. See the graph or the inequality solution, the driver should rent a car.

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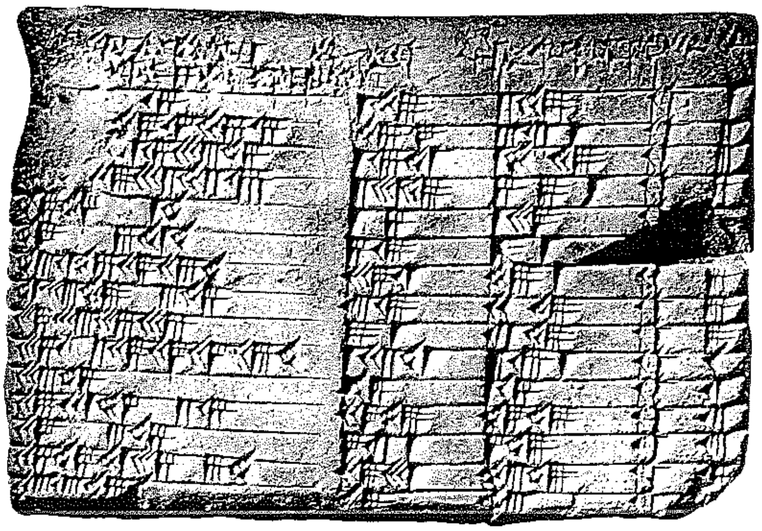
24/05/2016 – SUJET N°3

Pythagorean triples

A Pythagorean triple is a triple of whole numbers (a,b,c) where $a^2+b^2=c^2$. Below is an ancient Babylonian tablet listing fifteen Pythagorean triples. It is called Plimpton 322 (George Arthur Plimpton donated it to Columbia University).



Tasks :

1°) Justify the title "Pythagorean" triples. Give an example.

2°) Here is a list of triples :

$(a)\{5;12;13\}$ $(b)\{9;10;13\}$ $(c)\{8;15;17\}$ $(d)\{4;12;4\sqrt{10}\}$ $(e)\{30;40;50\}$

a) Justify which of the triples (above) are **not** Pythagorean triples.
b) Explain why the set of Pythagorean triples is or is not endless.

3°) A twin Pythagorean triple is a Pythagorean triple for which two values are consecutive numbers. Look at $\{5;12;13\}$ and $\{7;24;25\}$. In both triples, the largest numbers are consecutive numbers.

a) Let *m* and *n* be the two first numbers. Write the relation with m and n, so that $\{m;n;n+1\}$ is such a twin Pythagorean triple, and prove that *m* is necessary odd .(Hint : Let *a* be a whole number : if a^2 is odd, then *a* is odd).

b) Ask the jury for an odd number, and using the previous relation, compute the twin triple with this odd number as the smallest number.

4°) Find out all the Pythagorean triples with 3 consecutive numbers.

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24/05/2016 – CORRIGE SUJET N°3

Eléments de correction :

1°) The name is derived from the Pythagorean theorem, stating that every right triangle has side lengths satisfying the formula $a^2 + b^2 = c^2$; thus, Pythagorean triples describe the three integer side lengths of a right triangle.
Example : 3;4;5

2°) a) $(b)\{9;10;13\}$ and $(e)\{4;12;4\sqrt{10}\}$ are not Pythagorean triples because $13^2 \neq 9^2+10^2$ and $4\sqrt{10}$ is not an integer.
b) It's not endless, because, If you multiply each side by an integer, the result will be another triple.

3°) Let $\{m;n;n+1\}$ be a twin Pythagorean triple.
$m^2+n^2=(n+1)^2 \Leftrightarrow m^2+n^2=n^2+2n+1 \Leftrightarrow m^2=2n+1$. 2n+1 is obviously odd , thus m is odd.
Example : if the jury gives the number 37:
37^2 = 1369=2x684+1 then $\{37;684;685\}$ is a twin triple.

4°) $\{3;4;5\}$ is the only Pythagorean triple with 3 consecutive numbers.

$$(a-1)^2+a^2=(a+1)^2 \Leftrightarrow a^2-4a=0 \Leftrightarrow \begin{cases} a=0 & \textit{impossible} \\ a=4 & \textit{triple}\{3;4;5\} \end{cases}$$

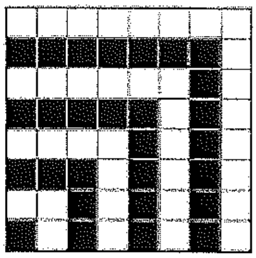
Proofs without words.

In mathematics, a proof without words is a proof of an identity or mathematical statement which can be presented as self-evident by a diagram without any accompanying explanatory text. When the diagram shows a particular case of a general statement, to be a proof, it must be generalisable.

<u>A proof without words for the sum of odd numbers theorem.</u>

$$1+3+5+7+...+(2n-1)=n^2$$

The statement that the sum of all positive odd numbers up to 2n-1 is a perfect square number, more precisely n^2 here, can be proved without words, using the diagram on the right. The first square consists in 1 block ; so 1 is the first square number. The next strip, composed of white blocks, shows how adding 3 more blocks makes another square number :4. The next strip, composed of black blocks, shows how adding 5 more blocks makes the next square number. This process can be continued indefinitely.



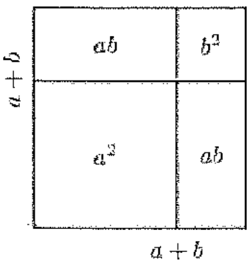
A proof without words for the sum of odd numbers theorem

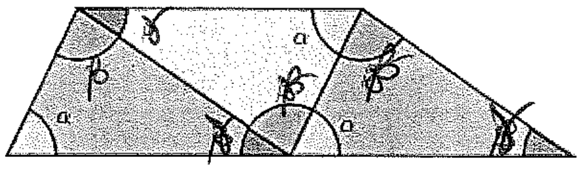
A proof without words for the area of a circle is given in the video.

Glossary: beads= perles

Tasks :

1. Present to the jury the proof *without words* of the sum odd numbers theorem.
2. Perform and explain to the jury the proof of the formula for the circle's area shown in the video.
3. Define the mathematical statements shown by the diagrams below.

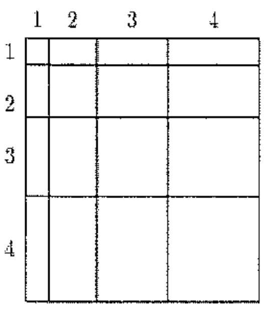


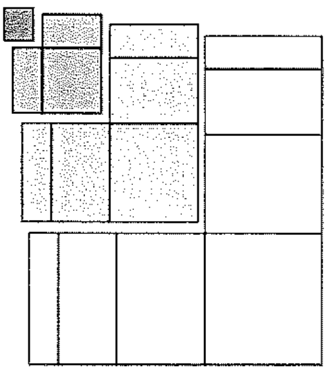


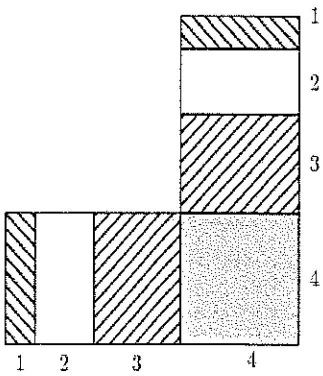
4. Using the diagrams below, prove to the jury the formula

$$(1+2+3+...+n)^2=1^3+2^3+3^3+...+n^3$$

Hint : explain why this "*right-angled elbow*" of the third diagram has total area equal to 4^3







Sources : *Adapted from Wikipedia* https://en.wikipedia.org/wiki/Proof_without_words
La géométrie de la chambre à air, J-P Boudine, éditions Vuibert
Des images pour les nombres, Kerléguer et Dumont, éditions du Kangourou

ANSWERS

Question 2.

The video transforms the circle of radius R into a triangle of base 2 pi R and perpendicular height R, which has area (2 pi R x R)/2 = pi R^2

Question 4.

Diagram n°1
Square is of side length 1+2+3+4, generalisable into 1+2+3+ … +n
This square has area $(1+2+3+4)^2$, which is the lefthand side of the statement.

Diagram n°2
Let's consider another point of view showing this side dissected into « right-angled elbows », for example the large one whose rectangles all share one dimension of 4.

Diagram n°3
These consist of 6 rectangles and 1 square of side length 4, called n°1.

Rectangles can be associated in pairs to obtain a new square of side length 4 according to the process given below.
- One rectangle of dimensions 4 and 2 can be added to the rectangle of dimensions 4 and 2 to obtain a square of side length 4, called n°2.
- One rectangle of dimensions 4 and 3 can be added to one rectangle of dimensions 4 and 1 to obtain a square of side length 4, called n°3.
- Two other rectangles of same dimensions allow to repeat this procedure to obtain a new square of side length 4, called n°4.

Therefore this « right-angled elbow » has a total area of $4 \times 4^2 = 4^3$.

The same procedure gives that the other « right-angled elbows » of this diagram have areas respectively of 3^3, 2^3 and 1^3.

Final argument.
The square of area $(1+2+3+4)^2$ is dissected into « right-angled elbows »
respectively of area 1^3, 2^3, 3^3 and 4^3, which gives the righthand side of the statement.

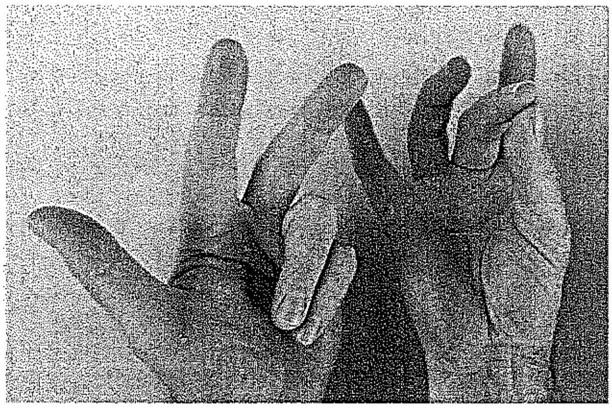
TASKS:

Watch and listen to the video "Why sixty".

1. From the video explain why number 60 was interesting for Babylonians.

2. Show the jury how to count to number 43 on your hands in the Babylonian way.

3. Indicate what number is shown on the picture below.



4. Nowadays numbers are given in base 10.
   For example, 5407 means $5\times10^3+4\times10^2+0\times10^1+7\times10^0$.
   a) Explain why counting to 43 with your hands in the Babylonian way is equivalent to counting in base 12.
   b) Write 2016 in base 12.

5. Find out remaining signs of base 12 in our everyday life.

Phalange Nuckle.

Eléments de correction :

1. Tout argument concernant l'abondance de diviseurs de 60 ou le fait de pouvoir compter sur ses doigts jusqu'à 60.
2. 3 douzaines et 7 phalanges. $(3\times 12 = 36) + (7\times 1) = 43$
3. 40 $3\times 12 + 4$ pha
4. $43=3\times 12^1+7\times 12^0$
5. $2016=12^3+2\times 12^2$
6. Les heures, le calendrier, la douzaine d'œufs…

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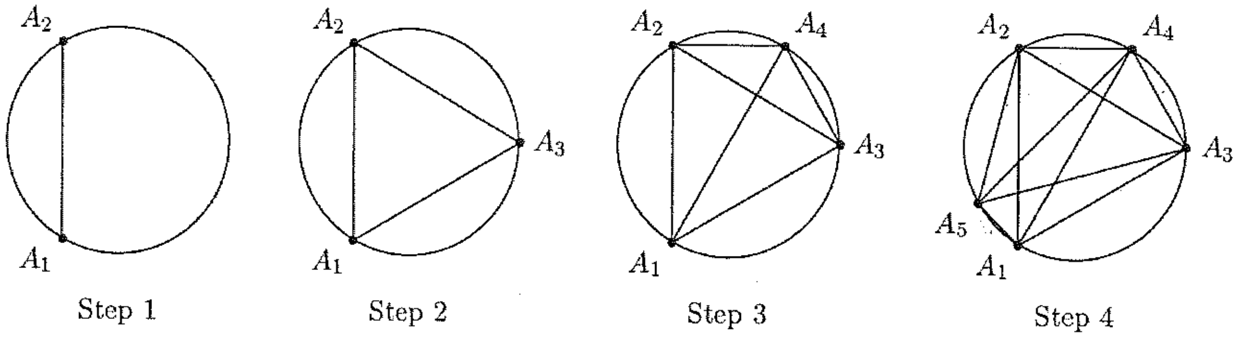
25/05/2016 – SUJET N°6

In the sequence of circle diagrams below, each point is joined to every other point on the circle by a straight line. Here $A_1A_2A_3$ is an equilateral triangle with $A_1A_2=1$.



<u>TASKS</u>

1 Perpendicular bisectors have been used to construct this sequence of points. Explain how and give a possible location for A_6 .

2 Define a formula connecting the number of line ℓ and the number of points p .

3 Assuming $A_2A_4=\frac{\sqrt{3}}{3}$, calculate the area of $A_1A_2A_3A_4$ specifying the steps of the calculation.

From: Higher GCSE Mathematics - D. Rayner -- ISBN:978-019-914-5744

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Session 2016 – Académie de Caen

25/05/2016 – CORRIGE SUJET N°6

Correction

1 For any $n \geq 2$, point A_{n+1} belongs to both the circle and the perpendicular bisector of line segment $A_{n-1}A_n$

One can locate A_6 on the circle, halfway between A_1 and A_3 .

2 One has $\ell = \frac{p(p-1)}{2}$: Each one of the p points is adjacent to every other (that is $p-1$ points). Dividing $p(p-1)$ by 2 we avoid counting edges twice.

3 Line segments A_1A_4 and A_2A_3 bisect each other perpendicularly, as a consequence A_1A_4 is a diameter (the perpendicular bisector passes through the centre of the circumcircle). Therefore $A_1A_2A_3$ is right angled at A_2 and Pythagoras' theorem gives

$$A_1A_4 = \sqrt{A_1A_2^2 + A_2A_4^2} = \sqrt{1 + \frac{1}{3}} = \frac{2\sqrt{3}}{3}$$

4 The area is half the product of the diagonals, that is

$$\frac{1 \times \frac{2\sqrt{3}}{3}}{2} = \frac{\sqrt{3}}{3}$$

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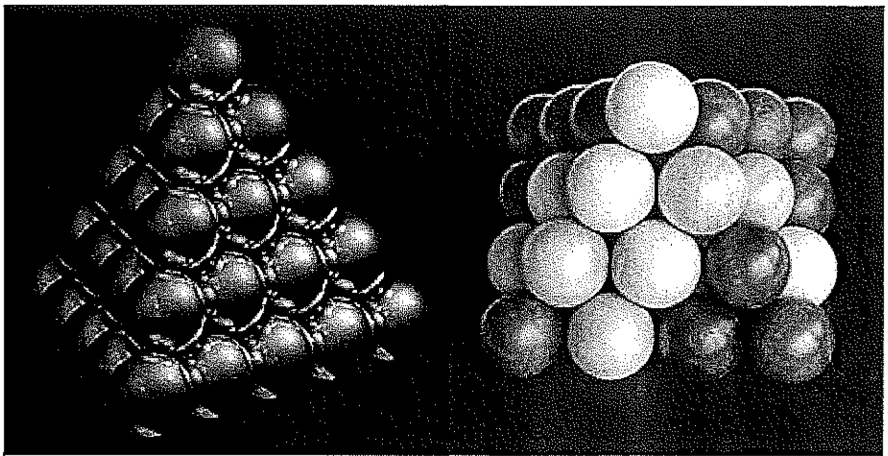
25/05/2016 – SUJET N°7

There are two ways to stack spheres into a pyramid. One (tetrahedral ball arrangement) is to start with a triangular base and build up the layers with successively smaller triangles. Another (square pyramid ball arrangement) is to start with a square base, adding layers of smaller squares on top until you reach the apex. (See image below.) Mathematically, both of these arrangements are the most efficient way to pack spheres in a three dimensional space.



tetrahedral ball arrangement
base length = 5
total number of balls = 35

square pyramid ball arrangement
base length = 4
total number of balls = 30

Source : http://www.had2know.com

Imagine that you were on a sailing ship a few hundred years ago and, in order to protect yourself from pirates, you had two cannons. Cannons need cannon balls and it is well known that the best way to stack cannon balls is to arrange them as pyramids.

1°) Stacking 10 cannon balls into a pyramid. Explain the idea.

2°) You decide to list all the numbers of cannon balls that can be stacked into pyramids. The mathematical genius of the ship says that he has already done it and gives you the table below.

Base length	1	2	3	4	5	6	7
Total number of balls in the base triangle	1	3	6	10	15		
Total number of balls in the tetrahedral pyramid	1	4	10	20	35		
Total number of balls in the square pyramid	1	5	14	30	55		

Complete the last two columns and explain your method.

3°) The Captain of the ship has a maximum of 100 cannon balls to stack. Stack these balls into 2 pyramids the best as possible. Explain.

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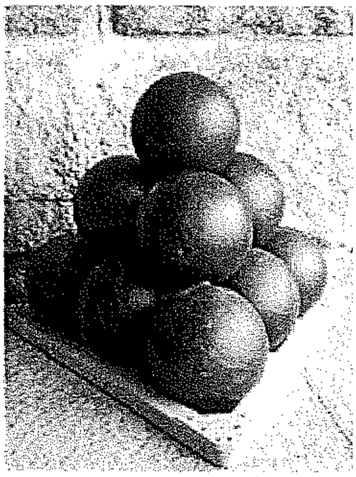
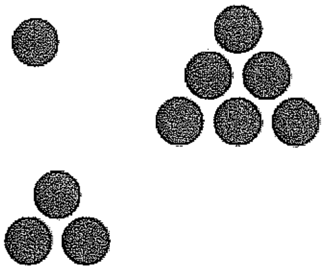
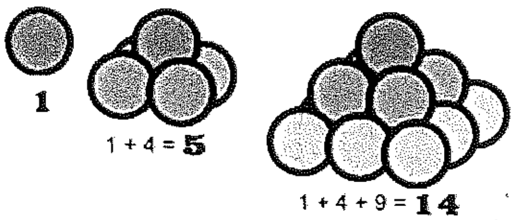
25/05/2016 – CORRIGE SUJET N°7

Eléments de correction :

1°) You can pile 10 cannon balls as a tetrahedral ball arrangement (1+3+6=10)
You cannot pile 10 cannon balls as a square pyramid ball arrangement (1+4=5 and 1+4+9=14)







2°)

Base length	1	2	3	4	5	6	7
Number of balls in the base triangle	1	3	6	10	15	21	28
Number of balls in the tetrahedral pyramid	1	4	10	20	35	56	84
Number of balls in the square pyramid	1	5	14	30	55	91	140

Number of balls in the base sq 1 4 9

2nd line : The sum of the numbers in the row above, from the beginning to the number just over it.
3rd line : Same way
4th line : The sum of the number just above and the previous one.

3°) 1 tetrahedral pyramid (14) and a square pyramid (84) = 98 balls
Square tetrahedral

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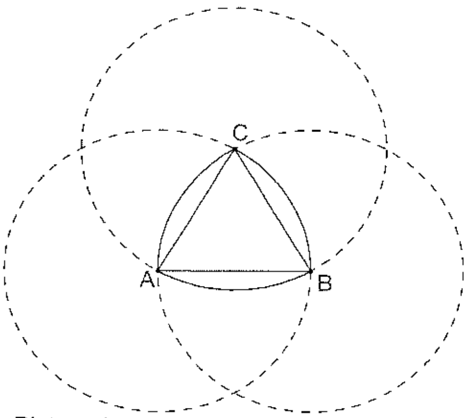
25/05/2016 - SUJET N°8

The Reuleaux triangle

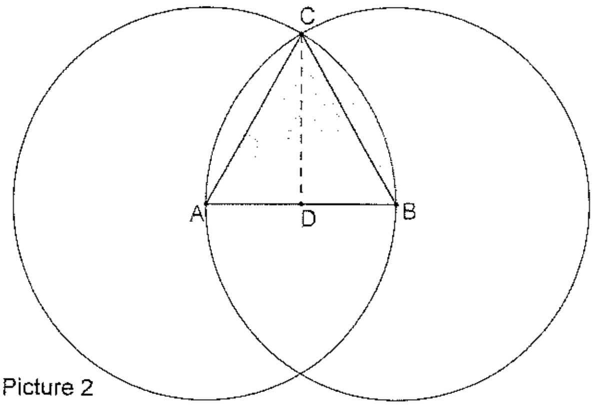
Franz Reuleaux (1829 – 1905) was a German scientist and engineer who is regarded as the founder of modern kinematics* and machine design. His name is mainly remembered today for an idea that would lead to some unexpected practical uses.
With a compass, draw three identical circles, each of radius r and each passing through tne center of the other two. The overlapping area is called the *Reuleaux triangle* (the « triangle » made by the circular arcs connecting A, B and C in picture 1).

Adapted from *Beautiful geometry,* by Eli Maor and Eugen Jost
Princeton University Press, Princeton and Oxford.

* Kinematics is the part of mechanics which describes movement.



Picture 1



Picture 2

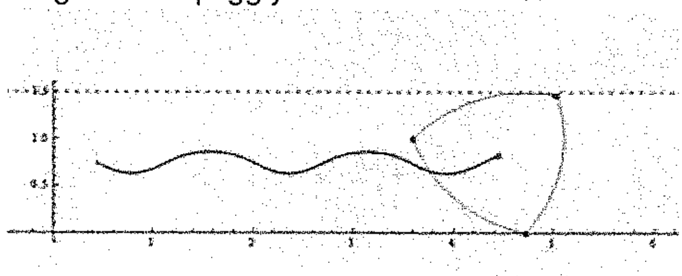
To facilitate pronunciation, refer to the Reuleaux triangle as « the R triangle » .

Tasks :

1. Describe the (ordinary) triangle ABC (sides, angles).
Suggest a point O that could be called the center of the Reuleaux triangle and explain how to construct this point.

2. Calculate the perimeter of the Reuleaux triangle in terms of r .

3. a. Describe the distance between a vertex of the Reuleaux triangle and any point on the opposite « side ».
b. Watch the video provided and explain how the Reuleaux triangle could be used as a « wheel » in contact with two parallel lines (picture below).
c. Explain the meaning of the squiggly line in the middle.



4. Calculate, in terms of r , the areas of the shaded circular sector in picture 2, of the ordinary triangle ABC, and finally of the Reuleaux triangle.

The Reuleaux triangle – answers

CORRIGE SUJET N°8

1. Each vertex of ABC is the center of one of the circles, and it lies on the other two circles. Hence $AB=BC=AC=r$, so the triangle ABC is equilateral and its three angles all measure 60°.
The center O of the circle that passes through the three points A, B, C could be called the center of the Reuleaux triangle. To construct it, draw the perpendicular bisectors of two sides of the triangle and name O their intersection point.

2. As the angles of the triangle ABC measure 60°, each of the circular arcs AB, BC, AC measures 1/6 of the complete circumference of a circle with radius r . So the perimeter of the Reuleaux triangle is 1/2 this circumference, so πr (equal to the circumference of a circle with *diameter* r).

3. a.The distance between any vertex of the Reuleaux triangle and the opposite « side » is equal to the radius r of the initial circles and the distance between the two parallel lines is also r .
b. The triangle can be moved along between the parallel lines, with always one vertex on one of the lines, while thye opposite « side » rolls on the other line.
The squiggly line shows the movement of the center of the Reuleaux triangle as it moves along between the two parallel lines.

4. The area of the shaded circular sector is 1/6 the area of a circle with radius r , so $\frac{\pi r^2}{6}$.

For the ordinary triangle ABC , let us use $AB=r$ as base and $CD=\frac{r\sqrt{3}}{2}$ as altitude.

So its area is equal to $\frac{r^2\sqrt{3}}{4}$.

As a consequence, the area of the small white portion between the segment BC and the right-hand-side circle is equal to $\frac{\pi r^2}{6}-\frac{r^2\sqrt{3}}{4}=\frac{2\pi r^2-3r^2\sqrt{3}}{12}=\frac{r^2(2\pi-\sqrt{3})}{12}$.

So the total area of the Reuleaux triangle is

$$A=\frac{\pi r^2}{6}+\frac{2\times r^2(2\pi-3\sqrt{3})}{12}=\frac{\pi r^2}{6}+\frac{r^2(2\pi-3\sqrt{3})}{6}=\frac{3r^2(\pi-\sqrt{3})}{6}=\frac{r^2(\pi-\sqrt{3})}{2}.$$

(Possible extension : The area of a circle with diameter r is equal to $\frac{\pi r^2}{4}$.

By using a calculator, we can check that $0,70<\frac{\pi-\sqrt{3}}{2}<0,71$ and $0,78<\frac{\pi}{4}<0,79$.

So, the area of the Reuleaux triangle is smaller than the area of a circle with diameter r even though they have the same perimeter. It could be expected because the Reuleaux lies entirely inside the circle with center O and diameter r .)