Académie de Caen - BACCALAUREAT – Session 2017

DNL Mathématiques/Anglais – SUJET N°1

Simpson's paradox

1) Watch the video "Simpson's paradox" (extract from "How statistics can be misleading" by Mark Liddell) and explain it to the jury. Explain what the "lurking variable" is.

2) Choose between situation A or B, and solve the problem :

Situation A : In the two-year period 1995-1996, the baseball player Derek Jeter had 195 hits in 630 at-bats (opportunities) and David Justice had 149 hits in 551 at-bats.

a. Which player had the higher batting average during this period (batting average is defined as hits divided by at-bats)?

b. Suppose we break the above data by year – we get the following table.

Determine who was the better hitter in 1995? And in 1996?

	1995		1996	
	Derek Jeter	David Justice	Derek Jeter	David Justice
Hits	12	104	183	45
At-bats	48	411	582	140

c. Explain how this example demonstrates Simpson's paradox.

Situation B :You and your friend decide to spend the whole weekend doing a quiz marathon. The winner is whoever gets the higher number of right answers out of a massive set of 1500 problems at the end of the 2 days.

- On the first day, you answer 1200 questions, out of which about 62.2% are right. Your friend answers 700 questions, out of which about 63.6% are correct. Your friend teases you over the phone about how he has got a higher rate of right answers at the end of the first day.
- On the second day, you answer 300 questions, out of which about 58.3% are correct. Your friend answers 800 questions, out of which about 58.8% answers are correct. Once again, your friend teases you because his percentage of right answers is greater.

Who wins this marathon? Explain why it's surprising.

3) In the early 1970s, the University of California, Berkeley was accused of gender discrimination over admission to graduate school. This table gives you the acceptance rate for each of the 6 departments.

Table 1: Data From Six Largest Departments of 1973 Berkeley Discrimination Case

Department	Men		Women	
	Applicants	Admitted	Applicants	Admitted
A	825	62%	108	82%
B	560	63%	25	68%
C	325	37%	593	34%
D	417	33%	375	35%
E	191	28%	393	24%
F	272	6%	341	7%

Source: Bickel, Hammel, and O'Connell (1975), table accessed via Wikipedia at https://en.wikipedia.org/wiki/Simpson%27s_paradox

Make a Simpson's paradox appear, and explain why they were accused of gender discrimination.

Académie de Caen - Baccalauréat général – Session 2017
Section européenne - DNL Mathématiques – Anglais
SUJET N°2

The game of Nim

Document 1 :

https://www.youtube.com/watch?v=Hof7P__P68I

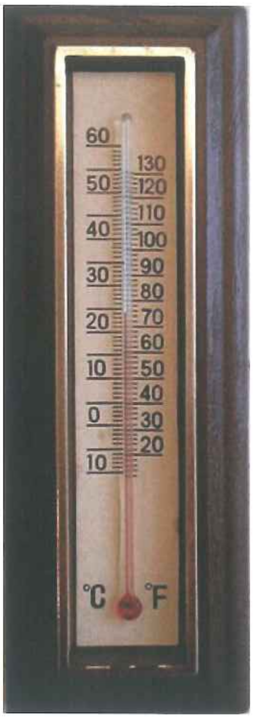
Tasks :

1) Explain the rules of the game to the jury.
2) Imagine it's your turn to play and you are facing 5 pieces. Explain how your opponent can be sure to win, no matter what you do.
3) What about facing 6, 7, 8 or 9 pieces?
4) Explain how to generalise to the case of facing n pieces (for any integer n).

Académie de Caen – Baccalauréat - Session 2017
DNL Mathématiques/Anglais - SUJET N° 3

Two scales of temperature

The Celsius (or centigrade) temperature scale, as part of the metric system, has been widely used by most countries since the 1970s. Before then, most western countries were using another scale of temperature: The Fahrenheit. Created in 1724 by German physicist Daniel Gabriel Fahrenheit, the Fahrenheit scale is still mainly used in the United States.



Wikipedia

Science-fiction amateurs may have read the novel *Fahrenheit 451* by Ray Bradbury (or seen the movie). A temperature of 451°F is the temperature at which paper catches fire spontaneously.

The Celsius-to-Fahrenheit conversion formula is described by a linear function. To define this function, one needs to know two reference points which provide both readings, in Celsius and Fahrenheit, of the same temperature. Those reference points are the change of state of pure water at standard atmospheric pressure:

	Celsius (°C)	Fahrenheit (°F)
Water freezing point	**0**	**32**
Water boiling point	**100**	**212**

From Wikipedia and various sources

TASKS

1. Explain the difference of meaning between the word “linear” (line 9) and its literal French translation “linéaire”.

2. Prove to the jury that the linear function which gives temperature F in Fahrenheit in relation to temperature C in Celsius is given by: $F = \frac{9}{5}C + 32$.

3. Using the function above, explain to the jury how to convert:
   - 20°C into Fahrenheit,
   - 451°F into Celsius.

4. Prove that there exists one temperature which gives the same readings in both scales.

5. Ana and Ben are talking:
   - Ben says: ' I know a trick to find easily an approximate conversion between both scales. *For any one-degree Celsius variation, there is almost a two-degree Fahrenheit variation*'.
   - Ana says: '*Not quite! You need to apply your trick from a known reference point*'.
     a. Explain Ben's assertion and Ana's correction to the jury.
     b. Perform this “trick” to the jury to convert:
   - 10°C into Fahrenheit,
   - 180°F into Celsius.

6. Give other physical concepts that are measured with two different units.

Two scales of temperature: Correction N°3

TASKS

1. The word "linear" means that the function is of the type: $f(x) = ax + b$. Its graph is a non-vertical line. The French corresponding translation is "affine". The French word "linéaire" means that the function is of the type: $f(x) = ax$. Its corresponding line goes through the origin of the coordinate system.

2. The function is of the type: $F(C) = aC + b$.
The slope is: $a = \frac{212-32}{100-0} = \frac{180}{100} = \frac{9}{5}$. Thus, $F(C) = \frac{9}{5}C + b$.
Knowing that $F(0) = 32$, one obtains $32 = \frac{9}{5} \times 0 + b \Leftrightarrow b = 32$. Therefore, the linear function which gives the temperature F in Fahrenheit in relation to the temperature C in Celsius is given by: $F = \frac{9}{5}C + 32$.

3. To convert:
   - $20°\text{C}$ into Fahrenheit: One plugs 20 into C. $F = \frac{9}{5} \times 20 + 32 = 68°\text{F}$
   - $451°\text{F}$ into Celsius: One plugs 451 into F and one solves the equation:
   $$451 = \frac{9}{5}C + 32 \Leftrightarrow 419 = \frac{9}{5}C \Leftrightarrow C = \frac{5 \times 419}{9} \approx 232.8°\text{C}$$

4. Let x be the common value.
$x = \frac{9}{5}x + 32 \Leftrightarrow x - \frac{9}{5}x = 32 \Leftrightarrow -\frac{4}{5}x = 32 \Leftrightarrow x = -\frac{5\times 32}{4} \Leftrightarrow x = -40$.
$-40°\text{C}$ and $-40°\text{F}$ represent the same temperature.

5. a. -Ben is referring to the slope of the line. $a = \frac{9}{5} = 1{,}8 \approx 2$.
The slope is the ratio of the y-values variation (Fahrenheit) over the corresponding x-values variation (Celsius). Therefore a variation of (almost) 2 degrees Fahrenheit gives a variation of 1 degree Celsius.
-However, because the line doesn't go through the origin of the coordinate system, one needs to apply this variation ratio from an existing point on the line. Hence Ana's correction.

   b.

   - From the reference point $0°\text{C} \leftrightarrow 32°\text{F}$, a variation of $10°\text{C}$ corresponds to a variation of $20°\text{F}$. Therefore $10°\text{C}$ corresponds to $32 + 20 = 52°\text{F}$ (exact value $50°\text{F}$)
   - From the reference point $100°\text{C} \leftrightarrow 212°\text{F}$, a variation of $212 - 180 = 32°\text{F}$ corresponds to a variation of $16°\text{C}$. Therefore $180°\text{F}$ corresponds to $100 - 16 = 84°\text{C}$ (exact value $82.2°\text{C}$)

6. No special knowledge is required for this question, but it can lead to a discussion with the candidate.
In thermodynamics, one uses the **Kelvin** (K) scale. Its null point is the absolute zero point, the temperature at which all thermal motion ceases. This is achieved at $-273.15°\text{C}$.
Therefore $°\text{C} = °K - 273.15$.
Distances in kilometers and miles.
Weights in kilograms and pounds....

Three math tricks that will blow your mind

Watch the video *"Three math tricks that will blow your mind"*: prepare the first two tricks and study the third one only if you have time.

Tasks

1. About the first trick *"Reading minds"* :
   a) Ask a member of the jury to choose a number between 1 and 20, and then perform the trick.
   b) Give a proof of this trick explaining how it works.
2. About the second trick *"Multiplying by 11"* :
   a) Perform this trick for the jury using number 24, then number 53.
   b) Explain what happens when using number 78, and give other examples of 2-digit numbers leading to a similar problem.
   c) Propose a solution to run the trick with 78.
   d) Give a proof of this trick, assuming that any 2-digit number, denoted tu, represents $10t + u$ and $11 = 10 + 1$.
3. About the third trick *"Fibonacci sequence"* :
   a) Comment on the name of the trick.
   b) Ask a member of the jury to choose two numbers less than 10, and then perform the trick.
   c) If time, give a proof of this trick.

Three math tricks that will blow your mind

Comments and answers

1. b) Let's denote n the given number. Here are the different steps of the trick :
   step 1: $n + 5$
   step 2 : $3 \times (n + 5) = 3n + 15$
   step 3 : $3 \times (n + 5) - 15 = 3n$
   step 4 : $(3 \times (n + 5) - 15) / 3 = 3n / 3 = n$
   QED !

2. a) $2 + 4 = \boxed{6}$, then $24 \times 11 = 2\boxed{6}4$; $5 + 3 = \boxed{8}$, then $53 \times 11 = 5\boxed{8}3$
   <u>Notice</u> *: the sum of the two digits gives the digit of the tens of the result*

   b) $7 + 8 = \boxed{15}$, but $78 \times 11 \neq 7\boxed{15}8$ since 78×11 is less than $100 \times 11 = 1100$…
   and so it is for any 2-digit number so that the sum of its digits is greater than or equal to 10.

   c) According to the two examples at question a), the sum of the two digits $7 + 8 = \boxed{15}$ should give the number of tens of the result.
   Therefore, 78×11 is equal to 7 hundreds + 15 tens + 8 ones, or (7 + 1) hundreds + 5 tens + 8 ones, which is 858.

   d) $$\begin{aligned}(10t + u) \times (10 + 1) &= 100t + 10t + 10u + 1u \\ &= 100t + 10(t + u) + 1u\end{aligned}$$
   which proves that the sum of the two digits gives <u>the number of tens</u> of the result.

3. a) The trick is actually not based on Fibonacci numbers but on numbers that are built with the same process : after choosing two initial numbers, all the following are the sum of the two previous ones.
   <u>Other keywords about Fibonacci</u> : Italy – 12^{th} /13^{th} century – Liber Abaci – sequence – rabbits

   b) <u>*Notice*</u> *:* choosing two numbers less than 10 must help (with a little chance !) to obtain a 2-digit number for the 7^{th} number, so that the multiplication by 11 seen before can be re-used !!!

   c) 1^{st} number : a
   2^{nd} number : b
   3^{rd} number : a + b
   4^{th} number : a + 2b
   5^{th} number : 2a + 3b
   6^{th} number : 3a + 5b
   7^{th} number : $\boxed{5a + 8b}$
   8^{th} number : 8a + 13b
   9^{th} number : 13a + 21b
   10^{th} number : 21a + 34b
   Sum of the 10 numbers : $55a + 88b = 11 \times (\boxed{5a + 8b})$ QED !

Académie de Caen - BACCALAUREAT - Session 2017

DNL Mathématiques/Anglais – SUJET N°5

A birthday trick

The magician asks the audience to do this calculation :

- *"Take the day of your birthday and multiply it by 13.*
- *Take the month of your birthday and multiply it by 40.*
- *Add these two numbers.*

Give me this total and I will find out your birthday !"

1. Which total do you get with your own birthday ? Explain the steps of the calculation.

2. How does it work ?

Euclidean division :

For every pair of natural numbers a and b, there are unique natural numbers q and r such that : $a=bq+r$ and $0 \leq r < b$

In the above equation :

- **a is the dividend**
- **b is the divisor**
- **q is the quotient**
- **r is the remainder**

a) Compute the remainder of the division of your total number by 13. Relate the remainder to your own birthday.

b) Let d be the day of your birthday, let m be the month of your birthday and let n be the number you get. Express n in terms of d and m .

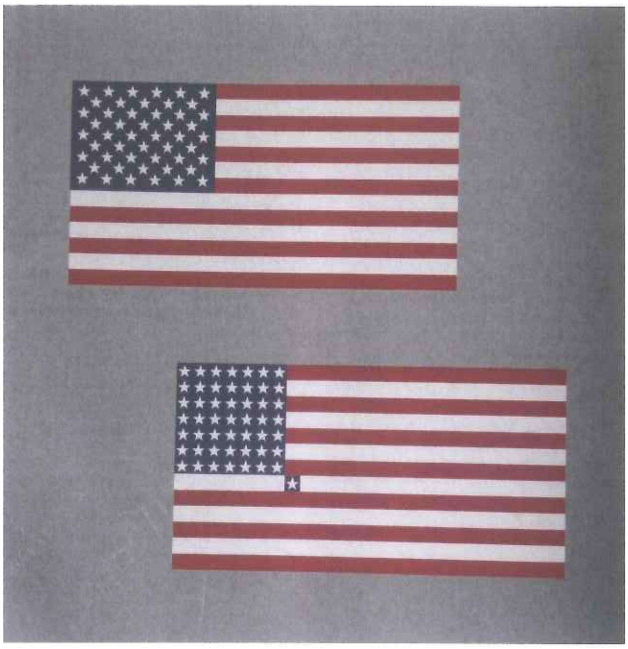
c) Explain why the remainder of the division n by 13 is m .

3. Find out one of the examiners' birthday (day and month).

Académie de Caen – Baccalauréat - Session 2017
DNL Mathématiques-Anglais – SUJET N°6

Number 50 and the American national flag

Let us take a look at 50: the number of stars that appear on the American national flag, the *Stars and Stripes*, each star representing one state of the Union.
Most people, looking at the pattern of the stars on the flag, will see an arrangement of nine horizontal rows, alternating between six and five stars per row. But, if you look at the star arrangement diagonally, an entirely new pattern emerges: five rows with 1, 3, 5, 7 and 9 stars, followed by the same pattern in reverse.
But 50 is also equal to 1^2+7^2, which can be arranged in a square of 49 stars and a single additional star anywhere outside. This single star could then stand for any one state of the Union, allowing each state to claim that it has a privileged status without offending any other state!



From *Beautiful Geometry*, by Eli Maor and Eugen Jost
Princeton University Press, Princeton and Oxford

Tasks:

1. Summarise the text in your own words, describe the picture and explain the connection between them.

2. Calculate the totals1+3, 1+3+5, 1+3+5+7, 1+3+5+7+9. Describe those sums and the pattern you notice in the totals. Make a link with one of the flags above.
Explain how many stars the flag would have if they were organised in 50 diagonals instead of 10.

3. Explain whether the following statement is true:
"*50 is the smallest integer that can be written as the sum of the squares of two positive integers in two different ways*".
Devise a method to find another integer that satisfies the same condition.

Baccalauréat DNL Mathématiques-Anglais
Session 2017 – Académie de Caen.

Number 50 and the American national flag

Elements de corrigé N°6

1. In the American flag, each star represents one of the 50 states than compose the United States of America.
In the normal American flag, if you count the stars horizontally, in 5 rows of 6 stars and 4 rows of 5 stars, you find $5\times6+4\times5=50$ stars; you can also count diagonally, in which case, you get: $1+3+5+7+9=25$ stars for the upper left half of the flag and $2\times25=50$ stars in total as well.
In the fictitious flag below, the stars have been organised in a square containing $7^2=49$ stars, plus an additional lonely star in the bottom right corner to keep a total of 50. Any individual state could claim that it is the one that is conspicuous in the corner, without offending the others that could also claim to be that particular star.

2. 1+3=4, 1+3+5=9, 1+3+5+7=16, 1+3+5+7+9=25
We add up the first two, three, four and five odd numbers. The totals are the squares of the 2, 3, 4 and 5. If the sum has five terms, then the total is five squared, etc.
If you look at the stars diagonally in the ordinary American flag, you can see five rows with 1, 3, 5, 7 and 9 stars, so a total of 25 stars in the upper left part of the flag. As the same pattern appears in the lower right part in the reverse order (9, then 7, 5, 3 and 1 stars), we check that there actually are 50 stars in total.

If there were 50 diagonals instead of 10, we would have to calculate the sum of the 25 first odd which is equal to 25^2 or 625, and the total number of stars would be the double of 625, so 1250.

3. The following statement is true:
"*50 is the smallest integer that can be written as the sum of the squares of two positive integers in two different ways*".
First of all, $50=5^2+5^2=7^2+1^2$. To check that 50 is the smallest integer that can be written as a sum of two squares, you can calculate the sums of the squares of two integers, until you reach a total of 50, you will find
$1^2+1^2=2$, $1^2+2^2=5$, $1^2+3^2=10$, $1^2+4^2=17$, $1^2+5^2=26$, $1^2+6^2=67$, $1^2+7^2=50$
$2^2+2^2=8$, $2^2+3^2=13$, $2^2+4^2=20$, $2^2+5^2=29$, $2^2+6^2=40$
$3^2+3^2=18$, $3^2+4^2=25$, $3^2+5^2=34$, $3^2+6^2=45$
$4^2+4^2=32$, $4^2+5^2=41$
$5^2+5^2=50$
50 is the only number that appears twice, so it actually is the smallest integer that can be written as the sum of the squares of two positive integers in two different ways.

The next integer that satisfies this condition is $65=1^2+8^2=4^2+7^2$.
You may just go on calculating the sums of squares until the same total appears twice.
Or you might try to find three integers such that $a^2+b^2=1+c^2$ or $a^2+b^2-1=c^2$ and you just check the successive values of a^2+b^2-1 until you get a perfect square.
However, not all the numbers that can be written as the sum of two squares in two different ways include the number 1: e.g. $85=6^2+8^2=2^2+9^2$.

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DNL Mathématiques/Anglais – SUJET N°7

The guidelines are just here to help you. Following them is not compulsory.

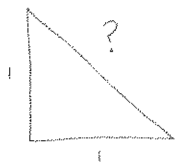
You can choose to follow any guidelines, in the order you want.

You are allowed to use your calculator.

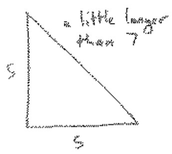
Not a number at all

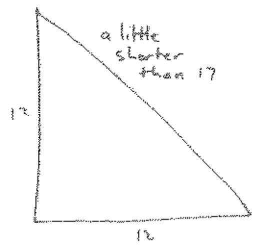
For us, the area of a circle with radius 1 is π. But two thousand years ago this was a vexing open question, important enough to attract the attention of Greek mathematicians.

Why was it so hard? For one thing, the Greeks didn't really think of π as a number, as we do. The numbers they understood were whole numbers (...). But the first great success of Greek geometry, the Pythagorean Theorem, turned out to be the ruin of their number system.



Here is a picture. In this right-angled triangle, the theorem says the square of the hypotenuse is $1^2 + 1^2 = 2$. In particular, the hypotenuse is longer than 1 and shorter than 2 (as you can check with your eyeballs, no theorem required). It can't be a whole number.





If we make the two legs of the triangle 5 units long, you can check with a ruler that the hypotenuse is just about 7 units long. Just about, but a bit too long... Or if you make the legs 12 units long, the hypotenuse is almost exactly 17 units, but is in fact a little shorter.

And at some point around the fifth century BC, a member of the Pythagorean School made a shocking discovery: there was *no way* to measure the isosceles right-angled triangle so that the length of each side was a whole number. To them, the length of that hypotenuse had been revealed to be *not a number at all.*

Adapted from The hidden maths of everyday life, Jordan Ellenberg

Vocabulary **vexing**: *épineux* **two legs:** *les deux côtés de l'angle droit*

The following guidelines may help you:

- You may recall the formula to calculate the area of a circle, and explain the assertion in the first sentence.
- What kinds of numbers other than whole numbers do you know?
- You may comment on the three pictures and explain why the hypotenuse cannot be 1 or 2 in the first case, neither exactly 7 in the second, nor exactly 17 in the third.
- You may try to find the exact values of these three hypotenuses. What kind of numbers do we obtain?

You may comment on the last paragraph of the text. Is « *not a number at all* » a good way of considering the situation, in your opinion? What can you say about the number π?

Académie de Caen - BACCALAUREAT – Session 2017

DNL Mathématiques/Anglais – ELEMENTS DE CORRECTION N°7

Answers.

1) Area of circle = pi R^2
since R=1 then Area = pi x 1^2= pi

2) integers, rational numbers, irrational numbers, real numbers, complex numbers

3) 4)
picture 1
applying Pythagoras theorem gives 1^2+1^2=2=d^2
then d = sqrt (2)
since 1<2<4 then 1< sqrt(2) < 2

picture 2
applying Pythagoras theorem gives 5^2+5^2=50=d^2
then d = sqrt (50) = 5 sqrt(2)
since 49<50<64 then 7< sqrt(2) < 8

picture 3
applying Pythagoras theorem gives 12^2+12^2=2 x 144=d^2
then d = 12 sqrt (2)
since 1< sqrt(2) < 2 then 12 < 12 sqrt(2) < 24

d is in the 3 cases above an irrational number, since not a whole number.

5) Number pi is an irrational number which means that its decimal equivalent never terminates and never repeats in cycles.

Other such examples are square roots of non perfect square whole numbers and of course also number e.

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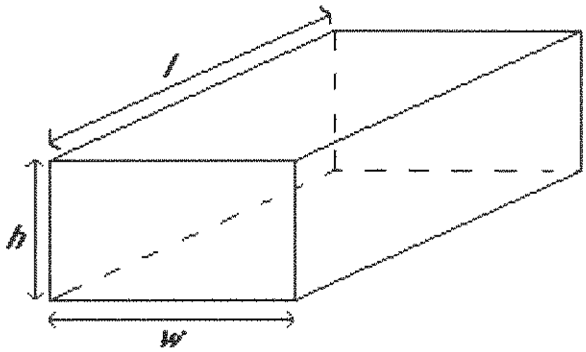
DNL Mathématiques / Anglais – SUJET N°8

Posting parcels

The Royal Mail Data Post International parcel service accepts parcels up to a maximum size as given in the rules below.

- The sum of the length, the height and the width must not exceed 90 cm.
- None of the length, height, width must exceed 60 cm.



Tasks

1. Explain which of the following parcels would be accepted for this service:

   (a) $\ell=62$ cm, $h=12$ cm, $w=15$ cm;

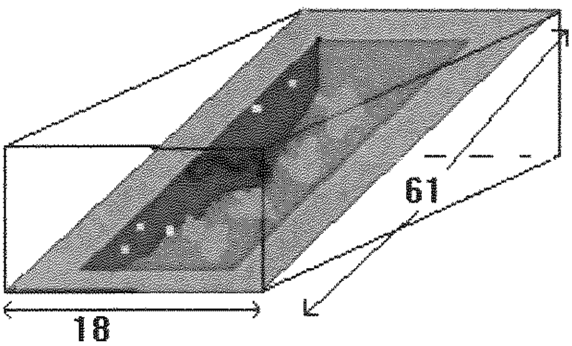
   (b) $\ell=50$ cm, $h=35$ cm, $w=15$ cm; (c) $\ell=55$ cm, $h=10$ cm,

   $w=15$ cm.

2. If the height is $h=10$ cm, prove that volume of the box is at most $V=10\times(80\,\ell-\ell^2)$.

   Define the dimensions of the rectangular box of maximum volume that can be sent through the Data Post service if the height is 10 cm?

3. A picture with frame, 61 cm by 18 cm, is to be placed diagonally in a rectangular box as shown below.



Is it possible to use a box with

- $\ell=6\ 2$ cm, $h=10$ cm, $w=18$ cm ? - $\ell=5\ 5$ cm, $h=17$ cm, $w=18$ cm ?

Find suitable dimensions for the box so that it would be accepted for the Data Post service.

4. Prove that, for any given height h , the volume is maximum when $\ell=w$.

Adapted from http://www.cimt.plymouth.ac.uk/resources/res1/postpar.htm

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DNL Mathématiques/Anglais – ELEMENTS DE CORRECTION N°8

1. Example (a) won't be accepted because the length is greater than 60cm

   Example (b) won't be accepted because the sum of the dimensions is greater than 90cm

   Example (c) will be accepted because none of the dimensions exceed 60cm and their sum is less than 90cm.

2. If the height is $h=10$ cm, the maximum width is $80-\ell$ since $\ell+h+w\leq 90$. The volume is the product of the dimensions $V=h\times\ell\times w=10\ell\times(80-\ell)$.

   Besides the other constraints give $0\leq\ell\leq 60$ and $0\leq 80-\ell\leq 60$, that is $20\leq\ell\leq 60$. Since V is a quadratic function in ℓ with a negative leading coefficient, the maximum is reached at $\ell=\dfrac{-b}{2a}=40$, its value is $10\times 40^2=16000\ cm^3$.

3. The first option is impossible since one dimension is greater than 60cm. For the second one, we have to check whether the diagonal of one side of the box is long enough. Using Pythagoras' theorem the problem boils down to

   $61^2\leq\ell^2+h^2$ that is $61^2\leq 55^2+17^2$ or $3721\leq 3314$

   which is false, therefore this one is impossible too. A possible answer is $\ell=60$, $h=12$ and $w=18$, in which case $\sqrt{\ell^2+12^2}\approx 61.19$ cm.

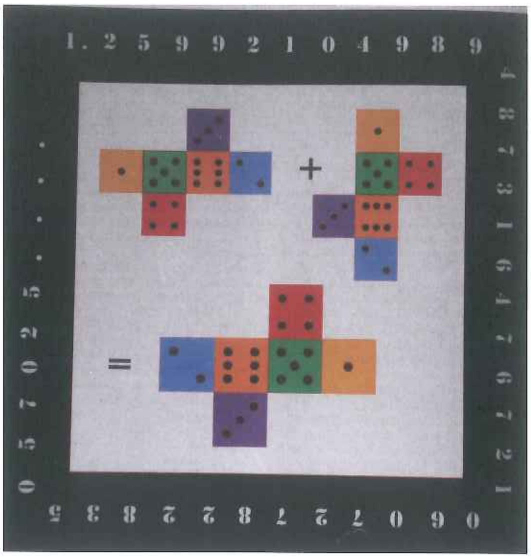
4. As in question 2, the volume is given by $V=h\times\ell\times(90-h-\ell)$, therefore the maximum is reached at $\ell=w=\dfrac{-b}{2a}=\dfrac{90-h}{2}$.

   (Possible extension) : Likewise one can prove that, for any given length ℓ, the maximum is reached when $h=w$, consequently the dimensions of the rectangular box of maximum volume that can be sent through the Data Post service are

   $\ell=h=w=\dfrac{900}{3}=30$ cm, in which case $V=30^3=27000\ cm^3$.

Doubling the cube

According to legend, at one time, the Greek town of Delos was afflicted by a devastating disease that nearly decimated its population. In desperation, the city elders consulted the oracles, who determined that the god Apollo was unhappy with the small size of the pedestal on which his statue was standing. To appease him, they recommended to double the volume of the cubical pedestal. The task was given to the town's mathematicians, who soon realized that doubling the *side* of the cube would not do it- it would increase the volume *eightfold* and would make the pedestal unreasonably large.



Adapted from *Beautiful geometry,* by Eli Maor and Eugen Jost
Princeton University Press, Princeton and Oxford.

Tasks :

1. Describe the situation and the picture in your own words, and suggest a connection.
Explain the last sentence of the text.

2. Explain whether the side of the cube should be multiplied by more or by less than 1.5 to double the volume of the cube.
Explain the connection between the problem and the equation $x^3=2$.

3. Considering the cubic function (defined by $f(x)=x^3$), explain why the problem has one perfect solution a.
As cube roots were unknown in Ancient Greece, find the approximation of a, rounded to the nearest hundredth, by trial and error.
Maybe it is now easier to connect the problem to the picture.

4. Decimal numbers were also unknown in Ancient Greece.
Guess and check: what is your best approximation of the solution of the equation $x^3=2$ by a fraction (with integer numerator and denominator)?

Baccalauréat DNL Mathématiques-Anglais
Session 2017 – Académie de Caen.

Doubling the cube
Eléments de correction N°9

Tasks :

1. The picture shows a plane figure showing three dice, which are of course cubes, with their six faces. The + sign suggests adding the first two cubes, which have the same size. The side of the third cube is somewhat longer. The equal sign shows that the volume of the larger cube is twice the volume of the smaller cube, like in the Delos legend. Around the colourful picture with the cubes, we can see white figures on a black background: 1.25992105 is the approximation of the cube root of 2 that a calculator gives. Here we have an approximation with 41 decimals.

2. If the original side of the cube is *s*, then its volume is $V=s^3$.
If you multiply the length of the sides by 1.5, the volume becomes $(1.5s)^3=3.375s^3=3.375V$ which is more than $2V$ so you should multiply the length of the sides by less than 1.5.
If we multiply the length of the sides by some number *x*, the volume will become $x^3\times V$. In order to double the volume, you must find a number *x* such that $x^3=2$.

3. The cubic function is continuous and increases all the time. As $1^3=1$ is smaller than 2, and $1.5^3=3.375$ is larger than 2, there is exactly one number *a* between 1 and 1.5 whose cube is exactly equal to 2.
Try $1.2^3=1.728$ and you know that 1.2 is too small. Likewise, 1.3 is too large, 1.25 is too small, and 1.26 is too large, 1.259 is too small and so on. The value of *a*, rounded off to the nearest hundredth is 1.26.

4. Suggestions: $\frac{5}{4}=1.25$, $\frac{126}{100}=\frac{63}{50}=1.26$, $\frac{19}{15}\approx1.267$, $\frac{34}{27}\approx1.259$