**BACCALAURÉAT - DNL Mathématiques/Anglais - Session 2017**
**Simpson's paradox – Sujet 1**

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1. Watch the video "Simpson's paradox" (extract from "How statistics can be misleading" by Mark Lidell) and explain it to the jury. Explain what the "*lurking variable*" is.

   `https://www.youtube.com/watch?v=sxYrzzy3cq8` (from 0'00 to 2'12)

2. Choose between situation A or B, and solve the problem:

   **Situation A:** In the two-year period 1995-1996, the baseball player Derek Jeter had 195 hits in 630 at-bats (opportunities) and David Justice had 149 hits in 551 at-bats.

   (a) Which player had the higher batting average during this period (batting average is defined as hits divided by at-bats)?

   (b) Suppose we break the above data by year - we get the following table.

| | 1995 | | 1996 | |
|---|---|---|---|---|
| | Derek Jeter | David Justice | Derek Jeter | David Justice |
| Hits | 12 | 104 | 183 | 45 |
| At-bats | 48 | 411 | 582 | 140 |

   Determine who was the better hitter in 1995. And in 1996?

   (c) Explain how this example demonstrates Simpson's paradox.

   **Situation B:** You and your friend decide to spend the whole week-end doing a quiz marathon. The winner is whoever gets the higher number of right answers out of a massive set of 1 500 problems at the end of 2 days.

   - On the first day, you answer 1 200 questions, out of which about 62.2% are right. Your friend answers 700 questions, out of which about 63.6% are correct. Your friend teases you over the phone about how he has got a higher rate of right answers at the ebd of the first day.
   - On the second day, you answer 300 questions, out of which about 58.3% are right. Your friend answers 800 questions, out of which about 58.8% are correct. Once again, your friend teases you because his percentage of right answers is greater.

   Who wins this marathon? Explain why it's surprising.

3. In the early 1970's, the University of California, Berkeley was accused of gender discrimination over admission to graduate school. This table gives you the acceptance rate for each of the 6 departments.

| | Male | | Female | |
|---|---|---|---|---|
| Department | Applicants | Admitted | Applicants | admitted |
| A | 825 | 62% | 108 | 82% |
| B | 560 | 63% | 25 | 68% |
| C | 325 | 37% | 593 | 34% |
| D | 417 | 33% | 375 | 35% |
| E | 191 | 28% | 393 | 24% |
| F | 272 | 6% | 341 | 7% |

   Source: Bickel Hammel and O'Connell (1975),
   `https://en.wikipedia.org/wiki/Simpson%27s_paradox`

   Make a Simpson's paradox appear, and explain why they were accused of gender discrimination.

# Document 1

https://www.youtube.com/watch?v=Hof7P__P68I

## Tasks

1. Explain the rules of the game to the jury.
2. Imagine it's your turn to play and you are facing 5 pieces. Explain how your opponent can be sure to win, no matter what you do.
3. What about facing 6, 7, 8 or 9 pieces?
4. Explain how to generalise to the case of facing $n$ pieces (for any integer $n$).



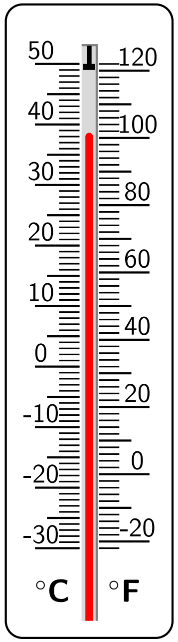
The Celsius (or centigrade) temperature scale, as part of the metric system, has been widely used by most countries since the 1970's. Before then, most western countries were using another scale of temperature: the Fahrenheit. Created in 1724 by German physicist Daniel Gabriel Fahrenheit, the Fahrenheit scale is still mainly used in the United States.

Science-fiction amateurs may have read the novel *Fahrenheit 451* by Ray Bradbury (or seen the movie). A temperature of 451°F is the temperature at which paper catches fire spontaneously.

The Celsius-to-Fahrenheit conversion formula is described by a linear function. To define this function, one needs to know two reference points which provide both readings, in Celsius and Fahrenheit, of the same temperature. those reference points are the change of state of pure water at standard atmospheric pressure.

| | **Celsius (°C)** | **Fahrenheit (°F)** |
|---|---|---|
| **Water freezing point** | **0** | **32** |
| **Water boiling point** | **100** | **212** |

*From Wikipedia and various sources*




## Tasks

1. Explain the difference of meaning between the word "linear" (line 9) and its literal French translation "*linéaire*".
2. Prove to the jury that the linear function which gives temperature $F$ in Fahrenheit in relation to temperature $C$ in Celsius is given by $F = \frac{9}{5}C + 32$.
3. Using the function above, explain to the jury how to convert 20°C into Fahrenheit and 451°F into Celsius.
4. Prove that there exists one temperature which gives the same readings in both scales.
5. Ana and Ben are talking

   **Ben:** "*I know a trick to find easily an approximate conversion between both scales. For any one-degree Celsius variation, there is almost a two-degree Fahrenheit variation.*"

   **Ana:** "*Not quite! You need to apply your trick from a known reference point.*"

   (a) Explain Ben's assertion ans Ana's correction to the jury.
   (b) Perform this "trick" to the jury to convert 10°C into Fahrenheit and 180°F into Celsius.
6. Give other physical concepts that are measured with two different units.

Watch the video "*Three math tricks that will blow your mind*": prepare the first two tricks and study the third one only if you have time.

`https://www.youtube.com/watch?v=-wkr_vf18cw`

## Tasks

1. About the first trick "*Reading minds*" (from 00:30 to 01:01)
   (a) Ask a member of the jury to choose a number between 1 and 20, and then perform the trick.
   (b) Give a proof of this trick explaining how it works.
2. About the second trick "*Multiply by 11*" (from 01:01 to 01:41)
   (a) Perform this trick for the jury using number 24, then number 53.
   (b) Explain what happens when using number 78, and give other examples of 2-digit numbers leading to a similar problem.
   (c) Propose a solution to run the trick with 78.
   (d) Give a proof of this trick, assuming that any 2-digit number, denoted $tu$, represents $10t + u$ and $11 = 10 + 1$.
3. About the third trick "*Fibonacci sequence*" (from 03:38 to 04:38)
   (a) Comment on the name of the trick.
   (b) Ask a member of the jury to choose two numbers less than 10, and then perform the trick.
   (c) If time, give a proof of this trick.



The magician asks the audience to do the following calculation:

- *"Take the day of your birthday and multiply it by 13.*
- *Take the month of your birthday and multiply it by 40.*
- *Add these two numbers.*

*Give me this total and I will find out your birthday!"*

1. Which total do you get with your own birthday? Explain the steps of your calculation.
2. How does it work?

   **Euclidean division**

   *For every pair of natural numbers $a$ and $b$, there are unique natural numbers $q$ and $r$ such that $a = bq + r$ and $0 \leq r < b$.*

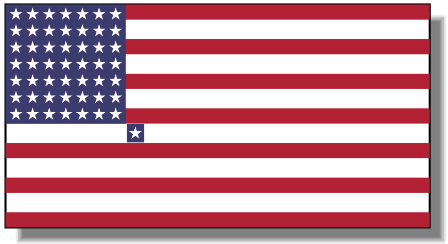
   In the above equation:

   - $a$ is the *dividend*
   - $b$ is the *divisor*
   - $q$ is the *quotient*
   - $r$ is the *remainder*

   (a) Compute the remainder of the division of your total number by $13$ Relate the remainder to your own birthday.
   (b) Let $d$ be the day of your birthday, let $m$ be the month of your birthday, and let $n$ be the total number you get. Express $n$ in terms of $d$ and $m$.
   (c) Explain why the remainder of the division $n$ by $13$ is $m$.
3. Find out one of the examiners' birthday (day and month).

**BACCALAURÉAT - DNL Mathématiques/Anglais - Session 2017**
**Number 50 and the American national flag – Sujet 6**

Let us take a look at 50: the number of stars that appear on the American national flag, the *Stars and Stripes*, each star representing one state of the Union.
Most people, looking at the pattern of the stars on the flag, will see an arrangement of nine horizontal rows, alternating between six and five stars per row. But, if you look at the star arrangement diagonally, an entirely new pattern emerges: five rows with 1, 3, 5, 7 and 9 stars, followed by the sale pattern in reverse. But 50 is also equal to $1^2 + 7^2$, which can be arranged in a square of 49 stars and a single additional star anywhere outside. This single star could then stand for any one sate of the Union, allowing each to claim that it has a privileged status without offending any other state!



From *Beautiful Geometry*, by Eli Maor and Eugen Jost
Princeton University Press, Princeton and Oxford

## Tasks

1. Summarise the text in your own words, describe the picture ans explain the connection between them.
2. Calculate the totals $1+3$, $1+3+5$, $1+3+5+7$, $1+3+5+7+9$. Describe those sums and the pattern you notice in the totals. Make a link with one of the flags above.

   Explain how many stars the flag would have if they were organised in 50 diagonals instead of 10.
3. Explain whether the following statement is true

   > *"50 is the smallest integer that can be written as the sum of the squares of two positive integers in two different ways."*

   Devise a method to find another integer that satisfies the same condition.

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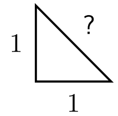
**Not a number at all – Sujet 7**

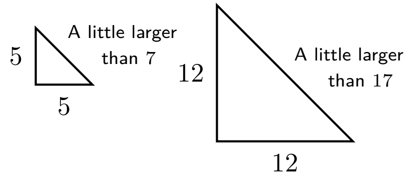
*The guidelines are just here to help you. Following them is not compulsory.*
*You can choose to follow any guidelines, in the order you want.*
*You are allowed to use your calculator.*

For us, the area of a circle with radius $1$ is $\pi$. But two thousand years ago this was a vexing[a] open question, important enough to attract the attention of Greek mathematicians.
Why was it so hard? For one thing, the Greeks didn't really think of $\pi$ as a number, as we do. The numbers they understood were whole numbers (...). But the first great success of Greek geometry, the Pythagorean Theorem, turned out to be the ruin of their system.




Here is a picture. In this right-angled triangle, the theorem says the square of the hypotenuse is $1^2 + 1^2 = 2$. In particular, the hypotenuse is longer than $1$ and shorter than $2$ (as you can check with your eyeballs, no theorem required). It can't be a whole number.

If we make the two legs[b] of the triangle $5$ units long, you can check with a ruler that the hypotenuse is just about $7$ units long. Just about, but a bit too long... Or if you make the legs $12$ units long, the hypotenuse is almost exactly $17$ units, but is in fact a little shorter.




And at some point around the fifth century BC, a member of the Pythagorean School made a shocking discovery: there was *no way* to measure the isosceles right-angled triangle so that the length of each side was a whole number. To them, the length of that hypotenuse has been revealed to *not a number at all.*

Adapted from *The hidden maths of everyday life*, Jordan Ellenberg

[a] épineux
[b] les deux côtés de l'angle droit

## The following guidelines may help you

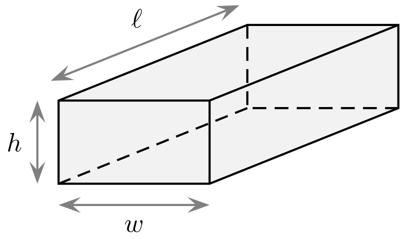
- You may recall the formula to calculate the area of a circle, and explain the assertion in the first sentence.
- What kinds of numbers other than whole numbers do you know?
- You may comment on the three pictures and explain why the hypotenuse cannot be $1$ or $2$ in the first case, neither exactly $7$ in the second, nor exactly $17$ in the third.
- You may try to find the exact values of these three hypotenuses. What kind of numbers do we obtain?

You may comment on the last paragraph of the text. Is "*not a number at all*" a good way of considering the situation, in your opinion? What can you say about the number $\pi$?

**BACCALAURÉAT - DNL Mathématiques/Anglais - Session 2017**
**Posting parcels – Sujet 8**

The Royal Mail Data Post International parcel service accepts parcels up to a maximum size as given in the rules below.

- The sum of the length, the height and the width must not exceed 90cm.
- None of the length, height, width must exceed 60 cm.




## Tasks

1. Explain to the jury which of the following parcels would be accepted for this service:

   (a) $\ell = 62$ cm, $h = 12$ cm, $w = 15$ cm;

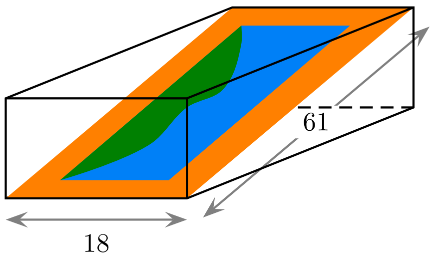
   (b) $\ell = 50$ cm, $h = 35$ cm, $w = 15$ cm;

   (c) $\ell = 55$ cm, $h = 10$ cm, $w = 15$ cm.

2. If the height is $h = 10$cm, prove that volume of the box is at most $V = 10 \times (80\ell - \ell^2)$.

   Define the dimensions of the rectangular box of maximum volume that can be sent through the Data Post service if the height is 10cm?

3. A picture with frame 61cm by 18cm is to be placed diagonally in a rectangular box as shown below.




   Is it possible to use a box with

   (a) $\ell = 62$ cm, $h = 10$ cm, $w = 18$ cm;

   (b) $\ell = 55$ cm, $h = 17$ cm, $w = 18$ cm;

   Find suitable dimensions for the box so that it would be accepted for the Data Post service.

4. Prove that, for any given height $h$, the volume is maximum when $\ell = w$.

Adapted from `http://www.cimt.plymouth.ac.uk/resources/res1/postpar.htm`

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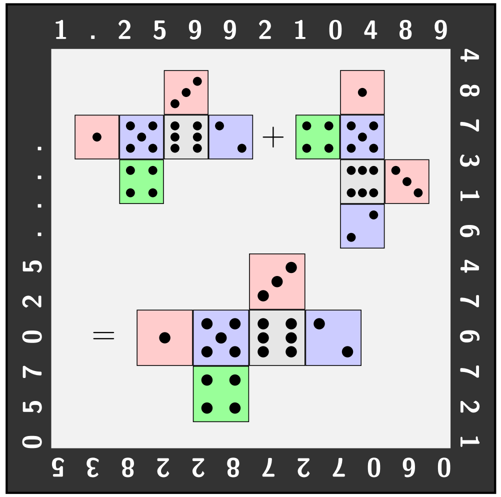
**Doubling the cube – Sujet 9**

According to legend, at one time, the Greek town of Delos was afflicted by a devastating disease that nearly decimated its population. In desperation, the city elders consulted the oracles, who determined that the God Apollo was unhappy with the small size of the pedestal on which his statue was standing. To appease him, they recommended to double the volume of the cubical pedestal. The task was given to the town's mathematician, who soon realized that doubling the *side* of the cube would not do it – it would increase the volume *eightfold* and would make the pedestal unreasonably large.




Adapted from *Beautiful Geometry*, by Eli Maor and Eugen Jost
Princeton University Press, Princeton and Oxford.

## Tasks

1. Describe the situation and the picture in your own words, and suggest a connection.

   Explain the last sentence of the text.

2. Explain whether the side of the cube should be multiplied by more or by less than $1.5$ to double the volume of the cube. Explain the connection between the problem and the equation $x^3 = 2$.

3. Considering the cubic function (defined by $f(x) = x^3$), explain why the problem has one perfect solution $a$.

   As cube roots were unknown in Ancient Greece, find the approximation of $a$, rounded to the nearest hundredth, by trial and error.

   Maybe it is now easier to connect the problem to the picture.

4. Decimal numbers were also unknown is Ancient Greece.

   Guess and check: what is your best approximation of the solution of the equation $x^3 = 2$ by a fraction (with integer numerator and denominator)?