

BACCALAURÉAT Session 2018 Académie de Caen DNL Mathématiques/Anglais

Cyclic numbers

Would you like to see a neat number trick? Take the number 142857 and multiply it by any number from 1 to 6. The result always has the same digits in the same order, if we say the first digit comes after the last. More generally, a cyclic number is positive integer in which cyclic permutations of the digits are successive multiples of the number.

Tasks

1. Explain to the jury what a cyclic number is.
2. Explain whether 076923 is a cyclic number or not.

Magic number 142857

Watch the video extract from the website "<http://www.numberphile.com>".

3. Perform this trick for the jury, asking them for a number n that is not a multiple of 7.
4. Explain what happens if n is a multiple of 7.
5. **Proof** - In the following, n is a whole number less than 10^6 .

- a. If n is a multiple of 7, say $n=7 \times q$, verify the following equality:

$$142857 \times 7q = 10^6 \times (q-1) + 10^6 - q$$

What if $q=1$ or 2?

Explain to the jury why this trick won't work for a multiple of 7.

- b. Writing $n=7 \times q+r$ (where r is in the range 1 to 6), explain why the trick works if n is not a multiple of 7.