BACCALAUREAT – DNL Mathématiques/Anglais – Session 2018

The guidelines are just here to help you. Following them is not compulsory.
You can choose to follow any guidelines, in the order you want.
You are allowed to use your calculator.

Mathematical mysteries: Hailstone sequences

Here's a little game to play.

Starting with any positive whole number *n,* **form a sequence** in the following way:
- If *n* is even, divide it by 2 to give *n*' = *n/2;*
- if n is odd, multiply it by 3 and add 1 to give *n'* = 3*n* + 1.

Then take *n*' as the new starting number and repeat the process.

The conjecture is that this algorithm always ends with 1, no matter what number you start with.

This apparently insignificant problem appeared for the first time in 1937 in the United States, invented by Lothar Collatz (1910 – 1990) at the university of Syracuse (not in Sicily but in the state of New York, USA).

A lot of researchers worked on this problem, mostly in the 60's (Collatz, Thwaites, Kakutani, Hasse, …). Some people even thought that it was a conspiracy by the Russians during the Cold War to slow down the American research.

Sequences formed in this way are sometimes called *hailstone sequences* because they go up and down, just like a hailstone in a cloud before crashing to Earth.

Adapted from https://plus.maths.org/content/mathematical-mysteries-hailstone-sequences (2014.12.09)

Hailstone: *grêlon*

The following guidelines may help you:

- Explain the sequence.
- Apply the algorithm starting with *n*=10.
- Try a few other numbers to check if the conjecture seems correct.
- Can you now explain why these sequences are called hailstone sequences?
- Who invented these sequences? Has anyone managed to prove the conjecture yet?
- This is the beginning of an algorithm that calculates the terms of the sequence until 1 is reached. Complete it.

```
Input n
While n ……
    If n is even then ……
    Else ……
    Disp n
```

Éléments de correction :

- *This sequence is called a hailstone sequence, because it goes up and down. If we start with 10, for example, the following numbers are 5, 16, 8, 4, 2,1, (4, 2, 1, 4, 2, 1 etc).*
- *This sequence was invented by Lothar Collatz in 1937, and nobody has yet been able to prove that it always ends with 1.*
- *If we start with 2, we get 1, and then 4, 2, 1 and so on. If we start with 4, we get 2, 1, 4, 2, 1 etc.*

Starting number	**9**	**15**	**42**	**100**
	28	46	21	50
	14	23	64	25
	7	70	32	76
	22	35	16	38
	11	106	8	19
	34	53	4	58
	17	160	2	29
	52	80	1	88
	26	40		44
	13	20		22
	40	10		11
	20	5		34
	10	16		17
	5	8		52
	16	4		26
	8	2		13
	4	1		40
	2			20
	1			10
				5
				16
				8
				4
				2
				1

- *The conjecture seems correct.*

- Input n
- While *n* > 1
- If n is even then n/2 → n
- Else 3n+1 → n
- Disp *n*