Behind the beehive

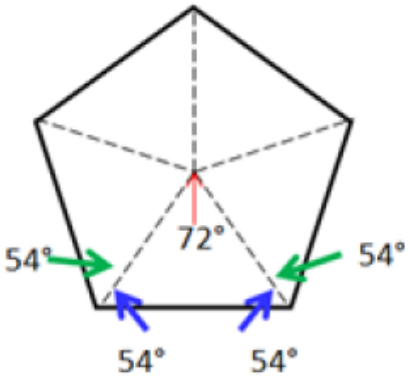
Tasks

1. Watch the video *"behind the beehive".*

2. Propose a short summary of the video and present the challenge honeybees have to face.

3. With the help of geometry, explain the three options that honeybees actually have to build honeycombs with regular pentagons.

4. a) The picture represents a regular pentagon.
Justify the different values of the angles.



b) Deduce why honeycombs cannot be built in a shape of regular pentagons.

5. a) Complete the following grid :

	Equilateral triangle with side *s*	Square with side *s*	Regular hexagon with side *s*
Perimeter P in terms of *s*	$P_3 = 3s$		
Area A in terms of *s*	$A_3 = \frac{\sqrt{3}}{4} s^2$		
Perimeter P in terms of A	$P_3 \approx 4{,}6\sqrt{A_3}$		

b) Conclude how mathematics explains why honeycombs are made of regular hexagons.

6. In the video (from 00:39 to 00:44), mathematics is said to be *"the universal language of all nature"*. Give examples using your own knowledge.

Vocabulary

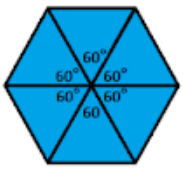
beehive : *ruche* ***raw instinct :*** *instinct naturel* ***wax :*** *cire* ***honeycomb :*** *structure alvéolaire d'une ruche*

Behind the beehive - *Comments and answers*

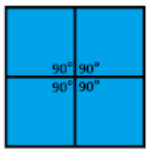
2. The point is that honeybees, everywhere in the world, always build honeycombs in the same hexagonal shape. The reason is not only due to biology as if bees were just programmed to build that shape, but has to be found out through a mathematical approach. Actually, the challenge for honeybees is to build a structure capable to store as much honey as they can, while using as little precious wax as possible.

3. Honeybees have three options to build honeycombs using regular shapes that fitted neatly :

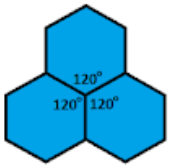
- with equilateral triangles because the angle at each vertex equals 60°, so that 6 equilateral can be placed from a common vertex and will cover the plane;



- with squares because the angle at each vertex equals 90°, so that 4 squares can be placed from a common vertex and will cover the plane;



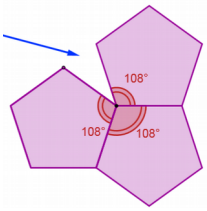
- with hexagons because the angle at each vertex equals 120°, so that 3 hexagons can be placed from a common vertex and will cover the plane.



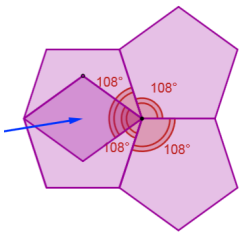
4. a) The regular pentagon is divided into 5 congruent isosceles triangles with a common vertex : the centre of the polygon. Therefore, each angle at the vertex equals 360 : 5 = 72°. Since each triangle is isosceles, the angles at the base are equal. Therefore, each of these angles measures (180 – 72) : 2 = 108 : 2 = 54°.

b) Honeycombs are not in a shape of regular pentagons, because it's not possible to cover the plane with regular pentagons without any gaps : since a regular pentagon has five interior angles of 108°, if we try to place pentagons around a point…

… we find that three must leave a gap because 3 × 108 = 324, which is less than 360° (a complete rotation around the common vertex)…



…and four must overlap because 4 × 108 = 432, which is more than 360°.



5.a)	Equilateral triangle with side *s*	Square with side *s*	Regular hexagon with side *s*
Perimeter P in terms of *s*	$P_3 = 3s$	$P_4 = 4s$	$P_6 = 6s$
Area A in terms of *s*	$A_3 = \frac{\sqrt{3}}{4} s^2$	$A_4 = s^2$	$A_6 = 6 \times A_3 = 6 \times \frac{\sqrt{3}}{4} s^2 = \frac{3\sqrt{3}}{2} s^2$
Perimeter P in terms of A	$P_3 \approx 4{,}6 \sqrt{A_3}$	$P_4 = 4 \sqrt{A_4}$	Since $s = \sqrt{\frac{2A_6}{3\sqrt{3}}}$, then $P_6 = 6 \times \sqrt{\frac{2}{3\sqrt{3}}} \sqrt{A_6} \approx 3{,}7 \sqrt{A_6}$

b) It appears that for the same area of storage of honey, the regular hexagon has the least perimeter and thus, will need less wax to build the structure.

6. Mathematics, *"the universal language of all nature"* : the number of petals of flowers or the number of spirals in a sunflower or a pine cone through the Fibonacci sequence; the radioactivity phenomenon with the exponential function; the shape of clouds, shores or human lungs (*poumons*) with fractals; forces explained with vectors; motion and gravity modeled with differential equations; areas and volumes calculated with integrals…