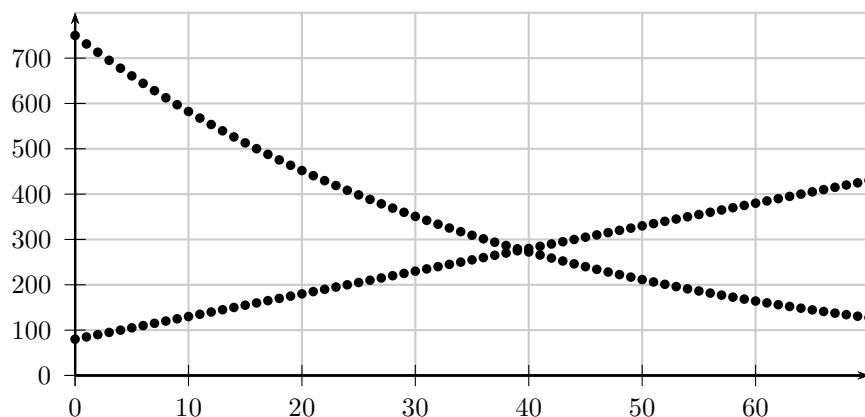




Detroit, the main city of Michigan became a symbol of the financial crisis. Since years 2000, its population decreases from 2.5% per year and in the same time 5000 houses are deserted. In 2013, the population of the city was 750 000 whereas the total amount of deserted houses was 80 000.

1. Compute the population of Detroit in 2014 and 2015.
2. Explain how to work out the general term of the sequence.
3. Give the formula of the sequence describing the number of deserted houses.
4. Match the two sequences with the graphs below.



5. Conjecture the year when the number of inhabitants will be less than the number of deserted houses.
6. In 2013, the price of a house in Detroit was \$68,000. A house loses 25% of its value each year. Give a method to determine when this house in Detroit will cost less than \$20.

Watch the video “Baseball”, extracted from the movie “Little big league”, 1994 (from 0’00 to 2’45)

<https://www.youtube.com/watch?v=pXtFSE7V1L0>

### Tasks:

1. Explain to the jury what the boy has to do for his homework.
  - (a) Three of the baseball players offer solutions of 15 hours, 8 hours and 4 hours. Explain how they obtain these solutions, and comment on the answers.
  - (b) One of the baseball players says “*Maybe this is trick question, and perhaps there is no answer, maybe it’s one of those trick questions*”. He is wrong. Why?
  - (c) What is the final calculation in the video?
3. Explaining the formula.
  - (a) Assume that it takes “ $a$ ” hours for one person to paint a house. What fraction of the house is painted in one hour?
  - (b) With the help of a second person needing “ $b$ ” hours, deduce the fraction of the house which is painted in one hour by the two people together.  
Then prove the formula  $\frac{a \times b}{a + b}$  used by the last baseball player.
  - (c) Compare with the answer given by the last baseball player.
4. Applying the formula. Together, Arthur and Ben can paint a wall in one hour. If it takes Arthur 3 hours working alone to paint it, how long does it take Ben working alone to paint this wall?
5. If time, what do you think about the formula involving 3 people?

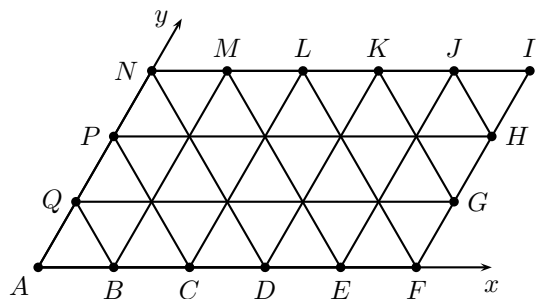
1. Watch the video (from 0' to 2'07), describe what's happening.

<https://www.youtube.com/watch?v=00ef3MHYEC0>

2. This problem can be turned into geometry:

$x$ -axis represents the volume contained in the 5 gallon jug

$y$ -axis the volume contained in the 3 gallon jug



Give  $x$  and  $y$ -coordinates for  $P$ ,  $J$  and  $F$ . Explain these coordinates by referring to the two jugs.

Which of these three points is a solution to the problem?

3. Explain what handling (with the jugs) is represented by the following paths

- (a)  $N$  to  $D$
- (b)  $Q$  to  $G$
- (c)  $L$  to  $C$

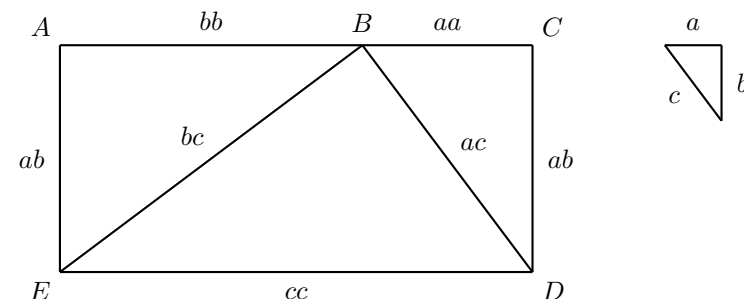
4. Present a solution to the problem starting with filling the 5 gallon jug.

5. Is it possible to get exactly 1 gallon using these two jugs?

1. Explain to the jury what you know about this famous theorem.
2. Explain the proof given in the video “*The Simplest Way To Prove the Pythagorean Theorem*”.

<https://www.bilibili.com/video/av3514210/>

3. Here is an extract from a website dedicated to the subject where another proof is given:



Create 3 scaled copies of the triangle with sides  $a$ ,  $b$ ,  $c$  by multiplying it by  $a$ ,  $b$  and  $c$  in turn. Put together, the three similar triangles thus obtained to form a rectangle whose upper side is  $a^2 + b^2$ , whereas the lower side is  $c^2$ .

Using angles  $\angle ABC$  and  $\angle AED$ , explain why  $ACDE$  is indeed a rectangle and conclude.

4. You can finish your presentation by telling the jury every detail you know about Pythagoras himself and his school.

### Sources

- Youtube channel “Explorium” (Dead?)  
<https://www.youtube.com/watch?v=NSd4A4Qqh50>
- The internet website “Cut The Knot”  
<https://www.cut-the-knot.org/pythagoras/>

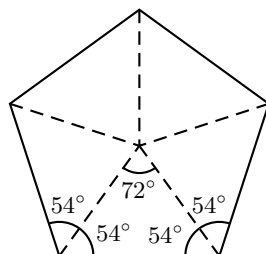
### Tasks

1. Watch the video “*behind the beehive*” (from 0'27 to 2'53), describe what's happening.

<https://www.youtube.com/watch?v=00ef3MHYEC0>

2. Propose a short summary of the video and present the challenge honeybees have to face.
3. With the help of geometry, explain the three options that honeybees actually have to build honeycombs with regular polygons.

4. (a) The picture represents a regular pentagon. Justify the different values of the angles.



- (b) Deduce why honeycombs cannot be built in a shape of regular pentagons.

5. (a) Complete the following grid:

	Equilateral triangle with side $s$	Square with side $s$	Regular hexagon with side $s$
Perimeter $P$ in terms of $s$	$P_3 = 3s$		
Area $A$ in terms of $s$	$A_3 = \frac{\sqrt{3}}{4}s^2$		
Perimeter $P$ in terms of $A$	$P_3 \approx 4.6\sqrt{A_3}$		

- (b) Conclude how mathematics explains why honeycombs are made of regular hexagons.
6. In the video (from 00:39 to 00:44), mathematics is said to be “*the universal language of all nature*”. Give examples using your own knowledge.

Watch the video [http://numberphile.com/videos/153\\_narcissistic.html](http://numberphile.com/videos/153_narcissistic.html)

1. Explain to the jury why 153 is a narcissistic number.
2. Give a four digit narcissistic number.
3. Explain why every one digit number is a narcissistic number.
4. According to the mathematician, is there an infinite list of narcissistic numbers?
5. Looking for a two digit narcissistic number  $n$ .
  - (a) Why is there no narcissistic two digit number with 0 for units digit? Let's call  $t$  the tens digit and  $u$  the units digit of  $n$ .
  - (b) Express  $n$  in terms of  $t$  and  $u$ .
  - (c) Prove that  $n$  is a narcissistic number, if and only if  $t$  is a solution of the equation:

$$x^2 - 10x + (u^2 - u) = 0 \quad (E)$$

- (d) Prove that there is no two digit narcissistic number. You may use the board below:

Value of $u$	Discriminant of $(E)$
0	100
1	100
2	92
3	76
4	52
5	20
6	-20
7	-68
8	-124
9	-188

6. Give a method to prove that 153 is the smallest 3 digit narcissistic number.

### Vocabulary

**beehive:** ruche

**wax:** cire

**raw instinct:** instinct naturel

**honeycomb:** structure alvéolaire d'une ruche

When someone takes a medical test, the result is said to be:

- a *true positive* if the test is positive and the person is infected;
- a *true negative* if the test is negative and the person is healthy;
- a *false positive* if the test is positive but the person is healthy;
- a *false negative* if the test is negative but the person is infected.

The false positive paradox is a statistical result where false positive tests are more probable than true positive tests, occurring when the overall population has a low incidence of a condition and the incidence rate is lower than the false positive rate.

The probability of a positive test result is determined not only by the accuracy of the test but by the characteristics of the sampled population.

So, in a society with very few infected people – fewer proportionately than the test gives false positives – there will actually be more who test positive for a disease incorrectly and don't have it than those who test positive accurately and do. The paradox has surprised many. Imagine running an HIV test on a population A, in which 1 person in 10,000 is infected. The test has a false positive rate of 0.0004 and no false negative rate. The expected outcome of a million tests on this population would be:

- 100 people would receive a true positive;
- 400 people would receive a false positive.

Only 100 of the 500 total people with a positive test result are actually infected. So, the probability of being infected after you are told the test is positive is only 20%, for a test that otherwise appears to be “over 99.95% accurate”.

Adapted from “False Positive Paradox”, Wikipedia

## Tasks

1. Explain in your own words the difference between a true and false positive test. Which of those two events is considered an error?
2. In the example, explain how to get the numbers 100 and 400 people receiving true and false positive tests. Is this situation a false positive paradox?
3. Why is the test said to be “over 99.95%” accurate?
4. Imagine you run a test having the same characteristics on a population B in which 200 out of 10,000 are infected. What is the expected outcome of a million tests on this population? Assuming your test is positive, determine the probability of being infected.
5. Explain the sentence “*The probability of a positive test result is determined not only by the accuracy of the test but by the characteristics of the sampled population*”.

Would you like to see a neat number trick? Take the number 142 857 and multiply it by any number from 1 to 6. The result always has the same digits in the same order, if we say the first digit comes after the last. More generally, a cyclic number is a positive integer in which cyclic permutations of the digits are successive multiples of the number.

## Tasks

1. Explain to the jury what a cyclic number is.
2. Explain whether 076 923 is a cyclic number or not.

**Magic number 142 857**

3. Watch the video extract from the website (from 0' to 2'17)

[http://www.numberphile.com/videos/cyclic\\_numbers.html](http://www.numberphile.com/videos/cyclic_numbers.html)

4. Perform this trick for the jury, asking them for a number  $n$  that is not a multiple of 7.
5. Explain what happens if  $n$  is a multiple of 7.
6. **Proof** – *In the following,  $n$  is a whole number less than  $10^6$ .*

(a) If  $n$  is a multiple of 7, say  $n = 7 \times q$ , verify the following equality:

$$142\,857 \times 7q = 10^6 \times (q - 1) + 10^6 - q$$

What if  $q = 1$  or  $q = 2$ ?

Explain to the jury why this trick won't work for a multiple of 7.

(b) Writing  $n = 7q + r$  (where  $r$  is in the range 1 to 6), explain why the trick works if  $n$  is not a multiple of 7.