BACCALAUREAT – DNL Mathématiques/Anglais – Session 2018

The Astronomical Unit (AU) is a unit of length used by Astronomers to describe solar system distances. One Astronomical Unit is the distance from the Sun to the Earth, and is approximately 150 million km.

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Distance from the Sun (in million km)	58	108	150	228	778	1430
Distance from the Sun (in AU)	0.39		1	1.52		

1°) Complete the table above with the distance in AU (with 2 significant digits).

In 1772 an astronomer named Johan Daniel Titius wrote to a colleague named Johan Bode that he had discovered a very interesting relationship between the distances of the planets from the Sun. Bode formulated the relationship as a mathematical expression in 1778. The formula, known as Bode's law relates the average distances of the planets from the sun to a simple mathematic progression of numbers :

Start with the initial list of numbers :	0	3	6	12	24			
Then, you get the sequence :	4	7	10	16	28			
And finally, to get Bode's law pattern :	0.4	0.7	1	1.6	2.8			

2°) Conjecture the pattern in the first row starting from 3. Explain how Bode got the following numbers. Calculate the next terms of Bode's law.

3°) When the law was originally published, only the 6 planets from Mercury to Saturn were known.
a) Check the values from Bode's law with the real distance from the Sun. Comment on your findings.
b) A large number of asteroids are about 433.8 million km from the Sun. What can you infer from this?

4°) Using Bode's law, work out the distance in AU where astronomers can expect to find the next farther planet.

At the beginning, the law was considered as interesting but of no great importance…. until the discovery of Uranus in 1781, which fits neatly into the series.

5°) Work out approximately the distance from the Sun to Uranus.

6°) In 1846, Neptune was discovered at an average distance of 30.1 UA. Apply Bode's Law, and comment.

Eléments de correction :

1°)

Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn
Distance from the Sun (in million km)	58	108	150	228	778	1430
Distance from the Sun (in AU)	0.39	0.72	1	1.52	5.19	9.53

2°) First row starting from 3 : multiply by 2
Second row : First row plus 4
Third row : Divide second row by 10.

Initial list of numbers	0	3	6	12	24	48	96	192
Sequence #2	4	7	10	16	28	52	100	196
Bode's law pattern	0.4	0.7	1	1.6	2.8	5.2	10.0	19.6

3°)

Planet	Mercury	Venus	Earth	Mars		Jupiter	Saturn	
Distance from the Sun in AU	0.39	0.72	1	1.52		5.19	9.53	
Bode's law	0.4	0.7	1	1.6	2.8	5.2	10.0	19.6

a) The law is satisfied by all the planets with a gap between Mars and Jupiter.
b) This gap could be justified by the large number of asteroids orbiting at
$$433.8 \times 10^6 \, km = \frac{433.8}{150} \simeq 2,9 \; AU$$
.
4°) The next term of Bode's law is 19.6.

5°) As it fits neatly into the sequence, we can work out that Uranus is $19.6 \times 150 = 2940$ million km from the sun. (Actually, it's about 2870 million)

6°) The next term of Bode's Law is 38.8 $\left(\frac{2 \times 192 + 4}{10} = 38.8 \right)$. It doesn't fit with the real distance.
Bode's law seems to be roughly correct for the first planets, but fails to give the right distances for the further ones.